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OPTIMAL CONTROL OF FLAPPING WING ACCOUNTING FOR NONSTEADY EFFECTS

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Abstract. The optimization of flapping wing trajectory for the minimization energy consumed by the wing and associated with induced power losses is considered in this paper. An aircraft with such a wing is assumed to move in a horizontal direction at constant velocity. The flapping wing moves up and down at constant velocity. Unsteady vortex wake influence is analyzed. Comparison of the efficiency coefficients is performed for the steady case, as well as for the one with the sinusoidal and optimal control for the flapping wing. Also, comparison with another type of the thrust creation (propeller) is performed.

Keywords: flapping wing, unsteady vortex wake, optimal control.

1. Introduction

Scientists are motivated to consider different problems in this field by the interest produced by flapping wing and aircraft with such wings. For example, K. D. Jones *et al*. have analyzed the wake structure behind plunging airfoils (Jones *et al*.1996). They have compared the picture obtained with the aid of the numerical method with the inviscid incompressible flow, and the experimental results and have shown that a plunging airfoil can produce drag, zero drag, or thrust depending on the motion parameters (reduced frequency and plunging amplitude). The thrust coefficient was obtained and it was noted that in the case of very small or very large reduced frequencies the numerical and the experimental results no longer coincide. It is because of the significant viscous influence in the first case and the flow separation in the second one. I. H. Tuncer and M. F. Platzer have compared the numerical results for the plunging airfoil obtained with the help of the different methods: with the in viscid incompressible flow, with the hybrid method with the Navier-Stokes equations solved in the boundary layer, and with viscous flow (Tuncer *et al*.1996). It was shown that these methods give close results (Re ~10^6). They also have investigated the questions of the efficiency of a plunging airfoil and of two airfoils in tandem interaction and their efficiency. I. H. Tuncer and M. Kaya have solved the optimization task for the maximization of thrust and efficiency for the airfoil in a combined plunge and pitch with the plunge and pitch amplitudes and phase shift between them as optimization parameters (Tuncer *et al*. 2004). Then they have solved a similar problem for the two airfoils in a biplane configuration (Tuncer *et al*. 2009). H. Nagai and T. Hayase have investigated the numerical and experimental aerodynamic characteristics and efficiency of an insect

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wing in forward flight (Nagai *et al*. 2009). G. J. Bermang and Z. J. Wang have considered the case of hovering insect flight and have found optimal wing kinematic that minimizes power consumption (Bermang *et al.* 2007). Z. J. Wang has sought the simplest efficient flapping motions with the aid of the model of quasi-steady forces and has made a comparison with steady forward flight (Wang 2008).

But up to now, in my opinion, there have not been enough studies concerned the question of the optimal kinematic of the flapping wing motion. This work therefore deals with this question; analytical and numerical investigations were made to determine flapping wing efficiency for a typical case. The results obtained were compared with the "traditional" type of thrust generation (propeller).

2. Model

The flight of an aircraft in a horizontal direction at constant speed and steady altitude is considered. The aircraft has a fixed wing for the production of lift and a flapping wing (or wings) only for the creation of thrust (Jones 2006). The flapping wing also creates lift at every moment of the motion, but the period-average lift is zero.

It is assumed that the wing is perfectly rigid (does not change its form under load) and that its mass and moment of inertia are zero (i.e., wing speed and its orientation in space can change instantly). It was shown earlier without taking the unsteady effect into consideration that the flapping wing should move in a straight line at constant velocity to minimize the amount of power consumed (Кравченко 2009). In this study, the flapping wing therefore moves up and down at constant velocity. It can also perform pitching motion. The oscillation amplitude and frequency are considered so that the vortex wake remains nearly flat. It was shown by J. Young, that there exists a region of the parameters of the flapping wing motion (reduced frequency and amplitude) where the Kutta condition is valid (Young 2005). So, assume that our wing parameters correspond to this region. This means that the flow is coming off the trailing edge of the wing without separation.

3. Evaluation of vortex wake influence

The problem in a simplified statement (the task of the model) has been considered to understand the main features of the processes taking place.

The wing is modelled with a bound vortex and two free vortices coming off the wings tips and closing to "rings" when the vertical velocity component is reversed (Fig 1). Assume that the distribution of circulation is constant along the wing.

It is well known, that the induced velocity V_i from the vortex section at a certain point of observation is given by the formula

$$V_i = \frac{\Gamma}{4\pi h}\left(\cos\theta + \cos\varphi\right)$$

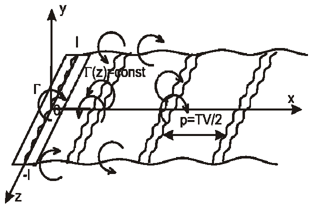

Fig 1. Simplified model for investigation of nonstationarity influence

where Γ is the circulation, *h* is the length of the perpendicular between the observation point and the line containing the vortex section, and θ and φ are the angles between the lines from the observation points to the ends of the vortex section and the line of the vortex section (Fig 2) (Лойцянский 2003).

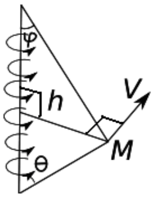

Fig 2. Vortex section

The analytical formula was found for the induced velocity created at a certain wing point *M*(*z*) at a certain moment of wing motion as the sum of the velocities generated by all the vortex sections:

$$\begin{aligned}
V_i^{sum}(x,z) &= \frac{\Gamma}{2\pi}\sum_{i=1}^{\infty}\frac{(-1)^i}{x+(i-1)p}\times \\
&\times\left(\frac{z}{\sqrt{(x+(i-1)p)^2+z^2}}+\frac{L-z}{\sqrt{(x+(i-1)p)^2+(L-z)^2}}\right)- \\
&-\frac{\Gamma}{4\pi p}\frac{x^2}{z}\frac{1}{\sqrt{x^2+z^2}}-\frac{\Gamma}{4\pi p}\frac{x^2}{L-z}\frac{1}{\sqrt{x^2+(L-z)^2}}+ \\
&+\frac{\Gamma}{4\pi z}\sum_{i=1}^{\infty}(-1)^i\times \\
&\times\left(\frac{x+(i-1)p}{\sqrt{(x+(i-1)p)^2+z^2}}+\frac{x+ip}{\sqrt{(x+ip)^2+z^2}}\right)+ \\
&+\frac{\Gamma}{4\pi(L-z)}\sum_{i=1}^{\infty}(-1)^i\times \\
&\times\left(\frac{x+(i-1)p}{\sqrt{(x+(i-1)p)^2+(L-z)^2}}+\frac{x+ip}{\sqrt{(x+ip)^2+(L-z)^2}}\right).
\end{aligned}$$

Since the above expression is too complicated for analytical investigation, numerical analysis was conducted for the detection of the main features.

Since the circulation value at the wing tips must always be equal to zero, let us specify the circulation function as

$$\Gamma=\Gamma_0,\ \varepsilon<z<L-\varepsilon,$$
$$\Gamma=0,\ 0<z<\varepsilon,\ L-\varepsilon<z<L,$$

where ε is a certain parameter, and *L* is wing span

(Лойцянский 2003). The following parameters were taken for calculation: $L=2m$; $\Gamma_0=1m^2/s$; $\varepsilon=0.01L$; $b=0.1L$; p – over the range from $0.5L$ to $10L$, where b is the wing chord.

Such are the main results. The induced force value averaged for a time period and for the wingspan was obtained. The vortices nearest the wing were found to give the main contribution. The induced velocities of the vortex wake generated on the wing alternate in direction forces, the first of them being positive (drag).

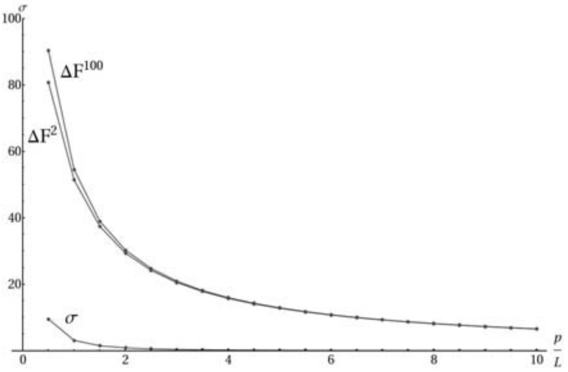

Fig 3. Comparison with the steady case

Comparison with the stationary case was made, the wing being modelled with a bound vortex and two infinite free vortices (Fig 3). Here, ΔF^{100} is the difference between the force in the steady case and the force from the first hundred vortex circular elements, ΔF^2 is the difference between the force in the steady case and the force from the first two vortex circular elements, and σ is the difference between ΔF^{100} and ΔF^2. As shown in the figure, neglecting all vortices and starting with the third one, gives an error of several percent if the value of the parameter p/L is not too small.

The force dependence on the wing position during the period is shown in figure 4:

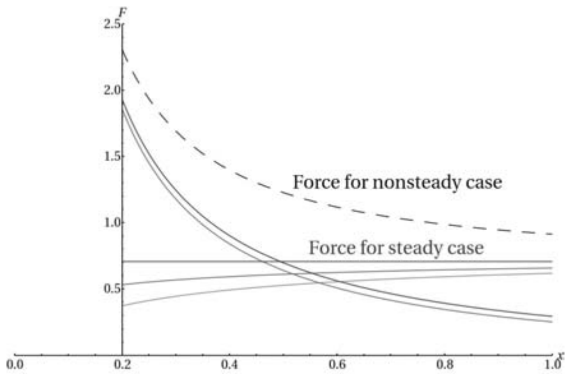

Fig 4. Plot of force vs. the wing position during the period

The horizontal line corresponds to the force in the steady case, and the dashed line corresponds to the nonsteady force. One can see that significant extra drag appears at the beginning of the period and then decreases fast, and during the rest of the time the movement is close to the stationary case.

4. Optimization task

It is clear that the instantaneous changing of the circulation value may not correspond to the optimal case, so for simplicity let us assume that the circulation distribution along the wing span is constant but suppose the circulation time dependence is a certain periodical function Γ(*t*) (Fig 5):

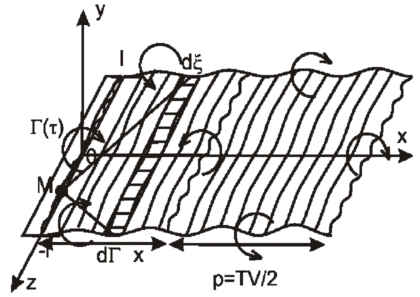

Fig 5. Vortex sheet investigated

At every moment of motion, the horseshoe vortex with circulation $d\Gamma = \dfrac{1}{V_\infty}\dfrac{\partial \Gamma}{\partial \tau} d\xi$ ($\tau=t-x/V_\infty$) leaves the wing. The span-averaged induced velocity generated with the vortex wake of length of 2*p* is:

$$v_i = \frac{1}{8\rho l V_\infty}\int_0^{2p}\Gamma_\tau'\int_{-l+\varepsilon}^{l-\varepsilon}\left[\frac{1}{x}\left(\frac{l-z}{\sqrt{x^2+(l-z)^2}}+\right.\right.$$

$$\left.+\frac{l+z}{\sqrt{x^2+(l+z)^2}}\right)+$$

$$+\frac{1}{l-z}\left(-\frac{x}{\sqrt{x^2+(l-z)^2}}+\frac{2p}{\sqrt{(2p)^2+(l-z)^2}}\right)+$$

$$\left.+\frac{1}{l+z}\left(-\frac{x}{\sqrt{x^2+(l+z)^2}}+\frac{2p}{\sqrt{(2p)^2+(l+z)^2}}\right)\right]dx.$$

The two vortices coming off the wing tips should also be taken into account. They induce the velocity:

$$u_i = \frac{\Gamma}{4\pi l}\int_{-l+\varepsilon}^{l-\varepsilon}\frac{dz}{l-z}.$$

The induced force is $F=\rho(v_i+u_i)\Gamma$, and the required power for the generation of this force is $W_i=FV_\infty$. The problem is to minimize the period-average induced power W_i, that is:

$$I_0 = \frac{1}{T}\int_0^T W_i dt = \frac{\rho V_\infty}{T}\int_0^T \Gamma(v_i+u_i)dt\,, \qquad (1)$$

provided that the power spent on the oscillatory motion is taken:

$$\rho V_\infty V_y \left(\int_0^{\frac{T}{2}} \Gamma dt - \int_{\frac{T}{2}}^{T} \Gamma dt \right) = \rho V_\infty V_y B = const. \qquad (2)$$

As we assume the flapping wing produces no lift, then the following condition must be taken into account:

$$\int_0^T \Gamma dt = 0 .$$

Let us expand the periodical circulation function in the Fourier series and determine the zero moment of time so that the sine function will only be present in this expression:

$$\Gamma(t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{2\pi nt}{T}\right). \qquad (3)$$

Assume an interchange of the operations of summation and integration can be done. By substituting expression (3) into (1) and (2) and carrying out some transformations, one can obtain:

$$I_0 = \frac{\rho V_\infty \ln\frac{2l}{\varepsilon}}{8\pi l} \sum_{n=1}^{\infty} a_n^2 + \frac{\rho Int V_\infty}{16lp} \sum_{n=1}^{\infty} n a_n^2 ,$$

$$Int = \int_0^{2p} \sin\left(\frac{\pi nx}{p}\right) \Phi(x,\varepsilon) dx ,$$

where Φ is the expression in the square brackets from the formula for v_i after integration with respect to z. Then, the optimization task can be reformulated:
minimize:

$$I = \frac{\ln\frac{2l}{\varepsilon}}{\pi} \sum_{n=1}^{\infty} a_n^2 + \frac{Int}{2p} \sum_{n=1}^{\infty} n a_n^2$$

under the condition:

$$\sum_{\tilde{n}=1}^{\infty} \frac{a_{\tilde{n}}}{\tilde{n}} = \frac{\pi B}{2T} ,$$

where the tilde over n means that the odd summand only appears in the sum, since the rest is nulled after the integration. The Lagrange function for this task is:

$$L = \frac{\ln(2l/\varepsilon)}{\pi} \sum_{n=1}^{\infty} a_n^2 + \frac{Int}{2p} \sum_{n=1}^{\infty} n a_n^2 + \lambda \left(\sum_{\tilde{n}=1}^{\infty} \frac{a_{\tilde{n}}}{\tilde{n}} - const \frac{\pi}{2\rho v_y V_\infty} \right),$$

where λ is the Lagrange coefficient.
Optimum conditions are thus:

$$\frac{\partial L}{\partial a_i} = 0, i \in [1,\infty), \frac{\partial L}{\partial \lambda} = 0 .$$

A really optimal solution can be obtained only if all the harmonics of oscillations are taken into account. This task is very difficult to solve and to analyze, however. Also, it is well known that the amplitudes of higher harmonics are usually rather small in comparison with the first ones. It was therefore decided to analyze the optimization problem only for a series of the first harmonics.

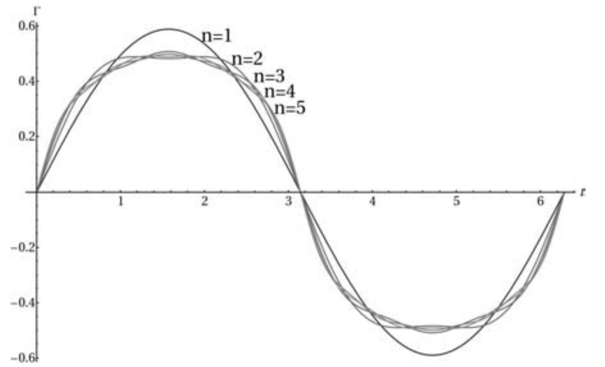

Fig 6. Function of optimal control obtained; n=1, 2, 3, 4, 5

The solution for the optimization task was found for a set of values of n for a set of values of C_0 (the constant *const* from condition (2)). The circulation function of optimal control is given in figure 6 for 1, 2, 3, 4, and 5 harmonics under conditions C_0=30 W. For n more or equal to 3, the graphics are rather close to each other (as was assumed above), so we may be content with the first several harmonics.

5. Determination of efficiency and comparisons

It is interesting to compare the gain with the optimal control of the wing in comparison with more often used and more easily implemented sinusoidal movement law (number of harmonics is 1). Let us determinate the efficiency as the rate of period-averaged useful power (i.e., the one suitable to wing thrust) to mean full power (i.e., the one spent on oscillatory motion):

$$W_T = \frac{1}{T} \int_0^T \rho \Gamma V_\infty (v_y + v_i + u_i) dt$$

$$W = \frac{1}{T} \int_0^T \rho \Gamma V_\infty v_y dt$$

$$\eta = \frac{W_T}{W} = 1 - \frac{\int_0^T \Gamma (v_i + u_i) dt}{v_y \left(\int_0^{\frac{T}{2}} \Gamma dt - \int_{\frac{T}{2}}^{T} \Gamma dt \right)} .$$

And after some computations, the formulas take this form:

$$\eta = 1 - \frac{\rho Int V_\infty}{16 p l C_0} \sum_{n=1}^{\infty} n a_n^2 - \frac{\rho V_\infty \ln(2l/\varepsilon)}{8\pi l C_0} \sum_{n=1}^{\infty} a_n^2 .$$

The numerical investigation was made for a set of parameters: l=1m, p=2l and p=l, and v_y=2m/s. The results are given in tables 1 and 2 for a set of values C_0; η_2 is the efficiency for the optimal control case when the optimization task is solved for n=2 (the circulation function includes two harmonics); η_3, η_4, and η_5 are defined similarly when n=3 and n=4; n=5; η_1 is the efficiency for the sinusoidal control.

Table 1. Numerical comparison of the efficiencies for optimal and sinusoidal control p/l=1

C_0,W	η_1	η_2	η_3	η_4	η_5
1	0.9937	0.9943	0.9943	0.9944	0.9944
10	0.9336	0.937	0.9377	0.9381	0.9381
20	0.8732	0.8797	0.8811	0.8817	0.8819
30	0.8129	0.8224	0.8245	0.8254	0.8257
40	0.7525	0.7651	0.7679	0.7690	0.7695
50	0.6921	0.7078	0.7113	0.7127	0.7133

Table 2. Numerical comparison of the efficiencies for optimal and sinusoidal control p/l=2

C_0,W	η_1	η_2	η_3	η_4	η_5
1	0.9909	0.9913	0.9914	0.9914	0.9914
10	0.8997	0.9042	0.9052	0.9056	0.9058
20	0.8086	0.8178	0.8191	0.8198	0.8201
30	0.7174	0.7301	0.7329	0.7339	0.7344
40	0.6262	0.6431	0.6467	0.6481	0.6488
50	0.5351	0.5560	0.5606	0.5623	0.5631

It is obvious that the increase in the number of harmonics to more than two practically does not produce any changes in the efficiency advantages. The utilization of optimal control gives little gain, and it becomes smaller as the motion period becomes longer.

The comparison of the efficiency for the steady case with sinusoidal control and with optimal control (n=5) was made. The plot of $\eta(T)$, where T is the wing thrust, is presented in figure 7.

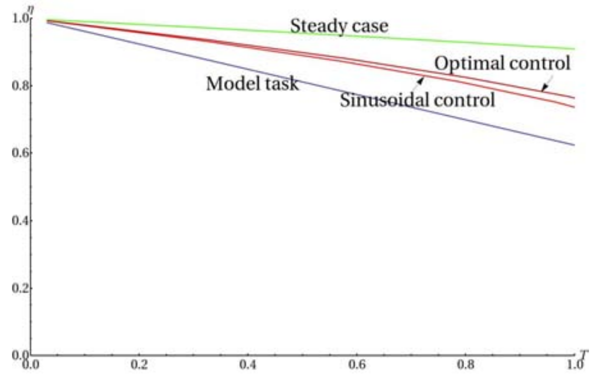

Fig 7. Efficiency versus wing thrust for steady and nonsteady cases

6. Comparison with ideal propeller

Assume the propeller will be taken to be the flapping wing equivalent if they are of the same swept area. Then, the equivalent propeller diameter is:

$$D = 4\sqrt{\frac{pl}{\pi}\frac{v_y}{V_\infty}} \tag{4}$$

The coefficient of the efficiency of the ideal propeller is defined (as described I. H. Tuncer *et al.*) (Tuncer *et al.* 2004):

$$\eta = \frac{1}{1+\dfrac{2T}{\rho V^2 \pi D^2}}$$

The efficiency of the ideal propeller is much more than the efficiency of the flapping wing, even for the steady case (Fig 8).

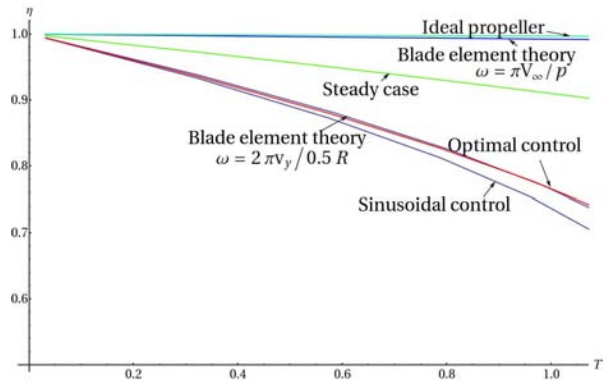

Fig 8. Efficiency comparison for p/l=2

7. Comparison with the propeller blade element theory

The quasilinear statement of the problem is used, i.e. the diameter of the stream from the propeller remains practically constant (Шайдаков *и др*. 1995). Assume the circulation of the bound vortex along the blade is constant. Then, the vortex sheet is the vortices coming off the blade tip and the hub. Let us define the angular velocity ω of the equivalent propeller (4) so that the rotary velocity for the characteristic section at 0.5R is equal to v_y:

$$\omega = \frac{2\pi v_y}{0.5R}. \tag{5}$$

The efficiency for such a propeller is

$$\eta = \frac{C_T \overline{V}}{m_K},$$

$C_T = \int_{r0}^{1} 8\overline{\Gamma}\overline{U}_1 d\overline{r}$, $m_K = \int_{r0}^{1} 8\overline{\Gamma}\overline{V}_1 \overline{r} d\overline{r}$ are the thrust and power coefficients of the propeller, $\overline{V}$ is the free stream relative velocity, $\overline{U}_1 = \overline{r} - \overline{u}_1$, $\overline{V}_1 = \overline{V} + \overline{v}_1$ are the relative velocity components of the real stream, $\overline{v}_1 = -\frac{\overline{V}}{2} + \sqrt{\left(\frac{\overline{V}}{2}\right)^2 + \overline{\Gamma}(1-\overline{\Gamma})}$, and $\overline{u}_1 = \frac{\overline{\Gamma}}{\overline{r}}$ are the axial and the peripheral components of the induced velocity. The velocities marked by bar are related to the blade tip rate ω*R*, $\overline{\Gamma} = \frac{\Gamma}{4\pi\omega R^2}$. The dependency of η(*T*) was obtained for the same parameters as for the flapping wing case (Fig 8). It is practically the same as the efficiency of the flapping wing (in the case of p/l=2). The dependency for p/l=1 is presented in figure 9:

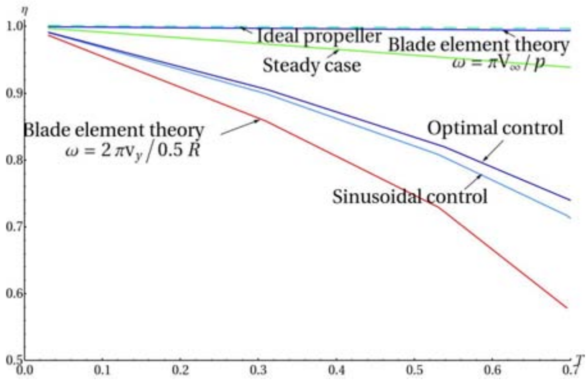

Fig 9. Efficiency comparison for p/l=1

As shown in the figure, the efficiency of the propeller is less than for the flapping wing one.

Also it should be mentioned that the results of comparison strongly depend on the method of determining ω (see (5)). If ω is defined so that the distances, which the vortex wakes, passes during the one swing and during the one propeller period are equal

$$\omega = \frac{\pi V_\infty}{p},$$

propeller efficiency appears higher for both parameters of p/l (Figs 8, 9). The results of the comparison are therefore not completely clear. Furthermore, the criterion of the comparison for the flapping wing in such a statement is not accurate, so the results presented for the propeller can be only qualitative.

8. Concluding remarks and future work

It is well known that constant circulation distribution is non-optimal regime both for the propeller and the wing. So, in the future, there are plans to solve a similar task for elliptical circulation distribution. It is possible that clearer results will be obtained. It should also be mentioned that we are taking into account only the power losses due to vortices. But viscous drag must also be taken into account in the analysis of efficiency.

Conclusions

1. An analytical model was proposed for the study of nonsteady effects.
2. The vortices nearest the wing were found to give the main contribution to the drag.
3. A method for the solution of the optimization problem was proposed based on the expansion of the characteristics in the Fourier series.
4. Optimal control laws were found for the different harmonics numbers. The utilization of optimal control gives little gain in comparison with sinusoidal circulation, and it becomes smaller as the motion period becomes longer.
5. Comparison with another variant of thrust creation was made. It was shown that the result of comparison strongly depends on the way ω is determined.

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NESTANDARTINIUS POVEIKIUS SUKELIANTIS OPTIMALUS PLASNOJANČIO SPARNO VALDYMAS

D. Kravchenko

S a n t r a u k a

Šiame tyrime svarstomas plasnojančio sparno trajektorijos optimizavimas tam, kad būtų minimizuota sparno sunaudota energija bei jos įtakoti galios nuostoliai. Manoma, kad orlaivis su tokiu sparnu juda pastoviu greičiu horizontalia kryptimi. Plasnojantis sparnas aukštyn ir žemyn juda pastoviu greičiu. Analizuotas nestacionaraus sūkurių pėdsako poveikis. Taip pat palygintas koeficientų efektyvumas stacionariu atveju bei esant sinusoidiniam ir optimaliam sparno valdymui.

Reikšminiai žodžiai: plasnojantis sparnas, nestacionarus sūkurių pėdsakas, optimalus reguliavimas.