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THE ANALYSIS OF GEOMETRICALLY NONLINEAR ELASTIC-PLASTIC SPACE FRAMES

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Abstract. The establishment of the real stress-strain state of the structure is one of the most important problems for designing and undertaking the reconstruction of building constructions as well as making calculations for the purpose of optimizing cross-sections of various structural elements. This task can be achieved by analysing the structure as a geometrically nonlinear system (refusing an assumption of small displacements) and taking into consideration plastic deformations. Modern computer technologies and mathematical tools enable us to perform strength analysis of space structures and to increase the accuracy of stress-strain state analysis. The present paper develops a technique for constructing a finite element tangent matrix for the nonlinear analysis of the space frame structure aimed at determining plastic deformations. The mathematical models of the problems based on static and kinematic formulations using the dual theory of mathematical programming were created for analysis. Strength conditions presented in construction codes and specifications AISC-LRFD and suggested by other researchers (e.g. Orbison's strength conditions) are used in the formulations of the analysed problems. The mathematical models of the considered problems are tested by calculating a two-storied space frame. The results of the performed analysis are compared with data obtained within the studies conducted by other researchers.

Keywords: elastic-plastic space structure, geometrical nonlinearity, tangent stiffness matrix, second–order analysis, MatLAB.

1. Introduction

The evaluation of the real stress-strain state of the structure is one of the most important problems of designing, reconstructing and optimizing calculations of the cross-sections of different structural elements. An acceptably accurate evaluation of the state can be obtained by analysing the structure as an elastic-plastic geometrically nonlinear system. For a long time, calculations have been made for structures considered to be geometrically linear systems behaving in the elastic or quasi–elastic phase. The results obtained from these calculations differ from the real behaviour of the structure. Recent editions of construction specifications (Eurocode 3 2006; STR 2.05.08:2005 2005) often include various nonlinear calculations. For example, it can be mentioned that the presently used definition "plastic hinge" or "plastic behaviour" is more frequently found not only in theoretical works (Atkočiūnas *et al.* 2008; Chen, Toma 1993; Ikrin 2005; G.-Q. Li, J.-J. Li 2007) but also in literature or handbooks describing construction standards and specifications (Trahair *et al.* 2008).

The discretization of the structure is another aspect of calculation that should be mentioned. Considering real objects, we may conclude that they are rather complicated space structures and may be divided into simpler elements, such as plane frames and separate beams or columns. The simplification of a structure in this way decreases computational accuracy connected with the general work of structure and actions of loads.

Powerful modern computer technologies enable us to perform strength analysis of space structures, which significantly increases the accuracy of deformed state analysis. Modern commercial programs of structure analysis widely used for design include nonlinear calculation subprograms and calculation kernels. Mathematical computation packages have been also developed, and thus enable us to solve various high-level mathematical problems one of which is associated with the analysis of geometrically nonlinear elastic-plastic space structures and can be solved by using the possibilities provided by computation packages (Jankovski, Atkočiūnas 2010; MathWorks Inc. 2010).

Modern computation packages allow us to develop a tangent stiffness matrix of the finite element of the space structure (Popov *et al.* 2010; Karkauskas 2007; Karkauskas, Popov 2009b) based on the regularities of tangent stiffness matrices of a general finite element (Karkauskas, Popov 2009a).

The problems of analyzing geometrically nonlinear elastic-plastic space structures were very popular among researchers in the last decade (Chiorean 2009; Chiorean, Barsan 2005; Kim, Kang 2002; Kim, Lee 2002; Kim *et al.* 2004, 2001; Van Long, Dang Hung 2008; Ngo-Huu *et al.* 2007; Richard Liew *et al.* 2000; Thai, Kim 2009). They developed and validated various types of second-order inelastic analysis for steel frames.

Research tasks are as follows:

1. To develop mathematical models for the dual formulation problem of analyzing geometrically nonlinear elastic-plastic space structures when strength conditions are written in terms of functional relations.
2. To develop mathematical models for analyzing the dual formulation problem of geometrically nonlinear elastic-plastic space structures when strength conditions are written in terms of linear functional relations.
3. To develop a technique for constructing a tangent stiffness matrix for a space finite frame element.
4. To consider strength conditions given for complex stress-strain state in construction codes and specifications and works of various researchers;
5. To estimate the stress-strain state of geometrically nonlinear elastic-plastic space structure based on the formulated analysis problems.
6. To perform numerical realizations and compare the derived results with the previously obtained data presented in works by other authors.

2. The problem of analyzing residual internal forces

As accepted in our research, displacements of element nodes are considerably larger than element dimensions. At the same time, an element form does not change and element strains remain small. Thus, we consider a problem when construction displacements are relatively large and strains are small. Thereby, we make an assumption that complementary energy is the convex function. An extreme energy principle (Čyras 1983; Čyras *et al.* 2004) was used to obtain residual internal forces of the elastic-plastic system in the real mode. This principle is formulated as follows:

„*Considering all statically admissible vectors of residual forces at the step to be considered, the actual vector is one for which the increment of complementary energy is minimum*“.

Additional strain energy is expressed as the square value of residual internal forces:

$$U_r = \frac{1}{2}\mathbf{S}_r^T \left[D_n\right]\mathbf{S}_r. \tag{1.1}$$

The vector of statically admissible residual internal forces, when a single load is applied to the structure, should satisfy the equation of equilibrium:

$$\left[A_n\right]\mathbf{S}_r = \mathbf{F} - \left[A_n\right]\mathbf{S}_e. \tag{1.2}$$

Strength conditions, in a general case, are expressed by the vector function:

$$\mathbf{f}\left(\mathbf{S}_e, \mathbf{S}_r, \mathbf{S}_0\right) \le 1. \tag{1.3}$$

Thus, the nonlinear mathematical programming problem, corresponding to the above formulated extreme energy principle, can be expressed as follows:

find

$$\min \ U_r = \frac{1}{2}\mathbf{S}_r^T \left[D_n\right]\mathbf{S}_r, \tag{2.1}$$

subject to

$$\left[A_n\right]\mathbf{S}_r = \mathbf{F} - \left[A_n\right]\mathbf{S}_e, \tag{2.2}$$

$$\mathbf{f}\left(\mathbf{S}_e, \mathbf{S}_r, \mathbf{S}_0\right) \le 1. \tag{2.3}$$

Further, this mathematical model will be referred to as the static formulation of the problem **analyzing residual internal forces**.

In such formulation, $\left[D_n\right]$ is $n \times n$ quasidiagonal flexibility matrix of the finite elements of the structure considering variations in the geometry of the structure; $\left[A_n\right]$ is $m \times n$ matrix of coefficients of equilibrium condition considering variations in the geometry of the structure; $\mathbf{F}$ is the vector of external forces applied to discrete model nodes of the structure; $\mathbf{S}_e$ is the vector of internal forces of an elastic response of the structure; $\mathbf{S}_r$ is the vector of residual internal forces of the structure; $\mathbf{S}_0$ is the vector of limiting internal forces of the structure; n is the number of internal variables of the forces; m is the degree of structure's freedom (DOF).

Problem (2.1)–(2.3) belongs to the class of convex mathematical programming problems where all the components of the strength function are convex functions.

3. The problem of analyzing residual displacements

The residual deformation state of an elastic-plastic system is determined based on the kinematic formulation of the analysed problem obtained using the duality theorem of mathematical programming. For dealing with this issue, the Lagrange function is suggested for problem (2.1)–(2.3). To express this function, the following multipliers were used: for strength conditions, nonnegative multipliers $\boldsymbol{\lambda} \ge \mathbf{0}$ were chosen while for static equations multipliers $\mathbf{u}_r$ (residual displacements) having any sign were used. Thus, the Lagrange function for extreme problem (2.1)–(2.3) is expressed as:

$$L\left(\mathbf{S}_r, \boldsymbol{\lambda}, \mathbf{u}_r\right) = \frac{1}{2}\mathbf{S}_r^T \left[D_n\right]\mathbf{S}_r + \boldsymbol{\lambda}^T\left(\mathbf{f}\left(\mathbf{S}_e, \mathbf{S}_r, \mathbf{S}_0\right) - 1\right) - \mathbf{u}_r^T\left(\mathbf{F} - \left[A_n\right]\mathbf{S}_e - \left[A_n\right]\mathbf{S}_r\right). \tag{3.1}$$

Constraints on the dual problem are stationary conditions of function 4 based on variables $\mathbf{S}_r$ (i.e. the first derivatives of the Lagrange function equated to zero) of the initial problem (2.1)–(2.3). In this case, stationary conditions of the Lagrange function are expressed as:

$$\frac{\partial L\left(\mathbf{S}_r, \boldsymbol{\lambda}, \mathbf{u}_r\right)}{\partial \mathbf{S}_r} = \mathbf{0} \tag{3.2}$$

or

$$\left[D_n\right]\mathbf{S}_r + \left[\frac{\partial \mathbf{f}\left(\mathbf{S}_e, \mathbf{S}_r, \mathbf{S}_0\right)}{\partial \mathbf{S}_r}\right]^T \boldsymbol{\lambda} - \left[A_n\right]^T \mathbf{u}_r = \mathbf{0}, \tag{3.3}$$

$$\boldsymbol{\lambda} \ge \mathbf{0}.$$

By transposing both sides of the equation, we can write:

$$\mathbf{u}_r^T\left[A_n\right]=\mathbf{S}_r^T\left[D_n\right]+\boldsymbol{\lambda}^T\left[\frac{\partial\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)}{\partial\mathbf{S}_r}\right]= \mathbf{S}_r^T\left[D_n\right]+\boldsymbol{\lambda}^T\left[\nabla\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right]. \quad (3.4)$$

The new Lagrange function obtained by substituting dependence $\mathbf{u}_r^T\left[A_n\right]$ into equation (3.1) is expressed as:

$$\begin{aligned}L\left(\mathbf{S}_r,\boldsymbol{\lambda}\right)\equiv&\frac{1}{2}\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_r-\boldsymbol{\lambda}^T\left(\mathbf{1}-\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right)+\\&\mathbf{u}_r^T\mathbf{F}-\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_e-\boldsymbol{\lambda}^T\left[\nabla\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right]\mathbf{S}_e-\\&\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_r-\boldsymbol{\lambda}^T\left[\nabla\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right]\mathbf{S}_r=\\&-\frac{1}{2}\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_r-\boldsymbol{\lambda}^T\left(\mathbf{1}-\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right)+\\&\mathbf{u}_r^T\mathbf{F}-\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_e-\\&\boldsymbol{\lambda}^T\left[\nabla\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right](\mathbf{S}_e+\mathbf{S}_r).\end{aligned} \quad (4)$$

Thereby, the objective function of the dual problem is the Lagrange function (4). The dual problem for (2.1)–(2.3) is a convex mathematical programming problem expressed as follows:

find

$$\max\ -\frac{1}{2}\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_r-\boldsymbol{\lambda}^T\left(\mathbf{1}-\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right)+\mathbf{u}_r^T\mathbf{F}- \mathbf{S}_r^T\left[D_n\right]\mathbf{S}_e-\boldsymbol{\lambda}^T\left[\nabla\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right](\mathbf{S}_e+\mathbf{S}_r), \quad (5.1)$$

subject to

$$\left[D_n\right]\mathbf{S}_r+\left[\nabla^T\mathbf{f}\left(\mathbf{S}_e,\mathbf{S}_r,\mathbf{S}_0\right)\right]\boldsymbol{\lambda}-\left[A_n\right]^T\mathbf{u}_r=\mathbf{0}, \quad (5.2)$$

$$\boldsymbol{\lambda}\geq\mathbf{0}. \quad (5.3)$$

Strength conditions in construction codes and specifications (AISC 2005; Eurocode 3 2006) are usually defined by linear functions. In a general case, these boundary conditions can be expressed as:

$$[\Phi](\mathbf{S}_e+\mathbf{S}_r)\leq\mathbf{S}_0, \quad (6)$$

where: $[\Phi]$ is $t\times n$ matrix of the coefficients of strength conditions; t is the number of strength conditions.

Then, the problem of residual internal forces can be expressed as:

find

$$\min\ U_r=\frac{1}{2}\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_r, \quad (7.1)$$

subject to

$$\left[A_n\right]\mathbf{S}_r=\mathbf{F}-\left[A_n\right]\mathbf{S}_e, \quad (7.2)$$

$$[\Phi]\mathbf{S}_r\leq\mathbf{S}_0-[\Phi]\mathbf{S}_e. \quad (7.3)$$

A kinematic formulation of this problem can be obtained as follows:

find

$$\max\ -\frac{1}{2}\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_r-\boldsymbol{\lambda}^T\mathbf{S}_0+\mathbf{u}_r^T\mathbf{F}-\mathbf{S}_r^T\left[D_n\right]\mathbf{S}_e, \quad (8.1)$$

subject to

$$\left[D_n\right]\mathbf{S}_r+[\Phi]^T\boldsymbol{\lambda}-\left[A_n\right]^T\mathbf{u}_r=\mathbf{0}, \quad (8.2)$$

$$\boldsymbol{\lambda}\geq\mathbf{0}. \quad (8.3)$$

The vectors of residual internal forces $\mathbf{S}_r$ and residual displacements $\mathbf{u}_r$ can be obtained using equations (7.2) and (8.2):

$$\begin{aligned}\mathbf{S}_r=&\Big(\left[D_n\right]^{-1}\left[A_n\right]^T\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left[A_n\right]\left[D_n\right]^{-1}-\\&\left[D_n\right]^{-1}\Big)[\Phi]^T\boldsymbol{\lambda}+\left[D_n\right]^{-1}\left[A_n\right]^T\times\\&\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left(\mathbf{F}-\left[A_n\right]\mathbf{S}_e\right)=\\&\left[\bar{G}\right][\Phi]^T\boldsymbol{\lambda}+\left[\bar{H}\right]\left(\mathbf{F}-\left[A_n\right]\mathbf{S}_e\right),\end{aligned} \quad (9)$$

$$\begin{aligned}\mathbf{u}_r=&\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left[A_n\right]\left[D_n\right]^{-1}[\Phi]^T\boldsymbol{\lambda}+\\&\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\mathbf{F}-\\&\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\mathbf{S}_e=\left[\bar{H}\right]^T[\Phi]^T\boldsymbol{\lambda}+\\&\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left(\mathbf{F}-\mathbf{S}_e\right),\end{aligned} \quad (10)$$

where:

$\left[\bar{G}\right]=\left[D_n\right]^{-1}\left[A_n\right]^T\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left[A_n\right]\left[D_n\right]^{-1}-\left[D_n\right]^{-1}$ is the influence matrix of residual internal forces;

$\left[\bar{H}\right]=\left[D_n\right]^{-1}\left[A_n\right]^T\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}$ is the influence matrix of residual displacements.

Then, expressions (9) and (10) are substituted into equation (8.1). Problem (8.1)–(8.3) is modified by performing appropriate mathematical operations. In this case, it is the extreme problem of the convex quadratic function in the orthant of nonnegative vector $\boldsymbol{\lambda}$:

find

$$\max\ \frac{1}{2}\boldsymbol{\lambda}^T[\Phi]\left[\bar{G}\right][\Phi]^T\boldsymbol{\lambda}-\boldsymbol{\lambda}^T\left(\mathbf{S}_0-[\Phi]\mathbf{S}_e\right)+ \boldsymbol{\lambda}^T[\Phi]\left[\bar{H}\right]\left(\mathbf{F}-\left[A_n\right]\mathbf{S}_e\right)+const \quad (11.1)$$

subject to

$$\boldsymbol{\lambda}\geq\mathbf{0}, \quad (11.2)$$

where:

$$const = \frac{1}{2}\mathbf{F}^T\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\mathbf{F} + \frac{1}{2}\mathbf{S}_e^T\left[A_n\right]^T\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left[A_n\right]\mathbf{S}_e - \mathbf{F}^T\left(\left[A_n\right]\left[D_n\right]^{-1}\left[A_n\right]^T\right)^{-1}\left[A_n\right]\mathbf{S}_e$$

is the member having no influence on the solution of problem (11.1)–(11.2), which in the case of the optimal problem solution, is equal to zero.

Problems (2.1)–(2.3), (5.1)–(5.3), (7.1)–(7.3), (8.1)–(8.3) and (11.1)–(11.2) are nonlinear convex mathematical programming problems with the variables represented by vectors $\mathbf{S}_r$, $\boldsymbol{\lambda}$ and $\mathbf{u}_r$. The solution to these problems allows us to determine real internal forces and displacement vectors, i.e. real stress-strain state using formulas:

$$\mathbf{S} = \mathbf{S}_e + \mathbf{S}_r, \quad \mathbf{u} = \mathbf{u}_e + \mathbf{u}_r. \tag{12}$$

4. The space frame element

The tangent stiffness method can be applied to solving the problem of the stress-strain state analysis of geometrically nonlinear elastic-plastic space structure (Karkauskas, Popov 2009a). For this purpose, the stiffness matrix of the finite element should be constructed.

The investigated element of the space frame structure is shown in Fig. 1. It is accepted that the nodes of the element can have displacements considerably larger than dimensions of the element; however, the form of the element does not change. Element strains remain small. The element is subjected to compression, tension and bending in two perpendicular planes as well as to torsion.

The vector of the nodal forces of the finite element in the local coordinate system consists of the following components (Fig. 1):

$$\mathbf{F}'_k = \left[F'_{ax} \quad F'_{ay} \quad F'_{az} \quad F'_{a\varphi_x} \quad F'_{a\varphi_y} \quad F'_{a\varphi_z} \quad F'_{bx} \quad F'_{by} \quad F'_{bz} \quad F'_{b\varphi_x} \quad F'_{b\varphi_y} \quad F'_{b\varphi_z}\right]^T.$$

The vector of nodal displacements of the finite space frame element is dual to the above introduced vector:

$$\mathbf{u}'_k = \left[u'_{k1} \quad u'_{k2} \quad u'_{k3} \quad u'_{k4} \quad u'_{k5} \quad u'_{k6} \quad u'_{k7} \quad u'_{k8} \quad u'_{k9} \quad u'_{k10} \quad u'_{k11} \quad u'_{k12}\right]^T.$$

This vector consists of horizontal, vertical, angular and torsional displacements. Positive directions of nodal forces and displacements are shown in Fig. 1.

All moments as well as angular and torsional displacements are denoted by double arrows following the right hand rule (Fig. 1).

The space frame finite element has twelve degrees of freedom in the local coordinate system. Linear and torsional displacements of any point of the element along and about local axis x' (element elongation or shortening) are described by linear functions $u'_{kx}(x)$ and $\varphi'_{kx}(x)$. Linear displacements along local axes y' and z' are described by nonlinear functions $u'_{ky}(x)$ and $u'_{kz}(x)$, respectively. Hermitian polynomials are usually used for approximating these functions (Barauskas 1998).

Hence, the vector of displacements for any point of the frame element

$$\mathbf{u}_k(x) = \left[u'_{kx}(x) \quad u'_{ky}(x) \quad u'_{kz}(x) \quad \varphi'_{kx}(x)\right]^T$$

can be expressed by the nodal displacements of the element as follows:

$$\mathbf{u}_k(x) = \left[N_k(x)\right]\mathbf{u}'_k, \tag{13}$$

where $\left[N_k(x)\right]$ is the matrix of Hermitian polynomials (Barauskas 1998; Barauskas *et al.* 2004).

Then, the total longitudinal and lateral deformations for the space frame element can be obtained in the following way (Čyras *et al.* 2004):

$$\begin{aligned}\boldsymbol{\varepsilon}_k(x) &= \frac{\partial u'_{kx}(x)}{\partial x} + \frac{1}{2}\left(\frac{\partial u'_{ky}(x)}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u'_{kz}(x)}{\partial x}\right)^2 - y\cdot\frac{\partial^2 u'_{kz}(x)}{\partial x^2} - z\cdot\frac{\partial^2 u'_{ky}(x)}{\partial x^2},\\ \boldsymbol{\gamma}_k(x) &= y\cdot\frac{\partial \varphi_{kx}(x)}{\partial x} + z\cdot\frac{\partial \varphi_{kx}(x)}{\partial x},\end{aligned} \tag{14}$$

where y and z are distances of the point from the neutral beam axis in the cross section of the element.

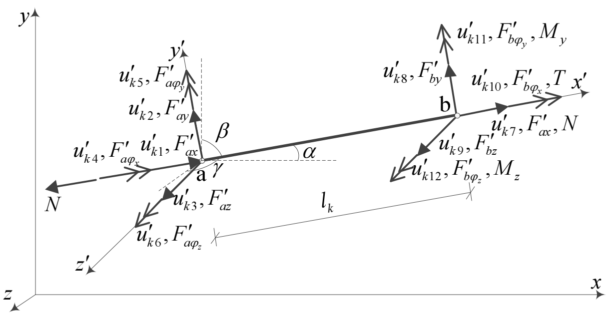

Fig. 1. Internal forces and nodal displacements of the space frame finite element in local coordinate system $x'y'z'$

The total longitudinal and lateral deformations expressed by nodal displacements can be obtained:

$$\begin{aligned}\boldsymbol{\varepsilon}_k(x) &= \left[C_5(x)\right]\mathbf{u}'_k + \frac{1}{2}\mathbf{u}'^T_k\left[C_{11}(x)\right]\mathbf{u}'_k + \frac{1}{2}\mathbf{u}'^T_k\left[C_{12}(x)\right]\mathbf{u}'_k - y\left[C_9(x)\right]\mathbf{u}'_k - z\left[C_{10}(x)\right]\mathbf{u}'_k,\\ \boldsymbol{\gamma}_k(x) &= y\left[C_8(x)\right]\mathbf{u}'_k + z\left[C_8(x)\right]\mathbf{u}'_k,\end{aligned} \tag{15}$$

where $\left[C_5(x)\right]$, $\left[C_8(x)\right]$, $\left[C_9(x)\right]$, $\left[C_{10}(x)\right]$, $\left[C_{11}(x)\right]$ and $\left[C_{12}(x)\right]$ are matrixes of the coefficients obtained by differentiating an appropriate row of the element in the matrix of Hermitian polynomials.

By substituting deformation expression (15) into the Hook's law $\boldsymbol{\sigma}_k(x)=E\cdot\boldsymbol{\varepsilon}_k(x)$ and $\boldsymbol{\tau}_k(x)=G\cdot\boldsymbol{\gamma}_k(x)$, we can obtain the expression of longitudinal and lateral stresses by nodal displacements.

Then, the expressions of stresses and deformations are substituted into the expression of the vector in light of the internal forces of the element (Karkauskas, Popov 2009a). As a result, the finite element tangent stiffness matrix is obtained:

$$\left[k_\tau'\right]=\left[k_e'\right]+\left[k_g'\right]+\left[k_u'\right], \tag{16}$$

where $\left[k_e'\right]$ is the stiffness matrix of small finite element displacements in the local coordinate system well-known and found in literature (e.g. Saouma 2000); $\left[k_g'\right]$ is the geometrical stiffness matrix of the finite element in the local coordinate system; $\left[k_u'\right]$ is the stiffness matrix of the initial finite element displacements in the local coordinate system.

To obtain a tangent stiffness matrix in the global coordinate system, a matrix of element direction cosines $\mathbf{T}_{ab}$ is constructed. Thus, the tangent stiffness matrix of a finite element in the global coordinate system is expressed as:

$$\left[k_\tau\right]=\left[T_{ab}\right]^T\left[k_\tau'\right]\left[T_{ab}\right], \tag{17}$$

where

$$\left[T_{ab}\right]=\begin{bmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \dfrac{-\cos\alpha\cos\beta}{\sqrt{\cos\alpha^2+\cos\gamma^2}} & \sqrt{\cos\alpha^2+\cos\gamma^2} & \dfrac{-\cos\beta\cos\gamma}{\sqrt{\cos\alpha^2+\cos\gamma^2}} \\ \dfrac{-\cos\gamma}{\sqrt{\cos\alpha^2+\cos\gamma^2}} & 0 & \dfrac{\cos\alpha}{\sqrt{\cos\alpha^2+\cos\gamma^2}} \end{bmatrix}$$

is the matrix of direction cosines for any nonvertical finite element (Saouma 2000); $\left[T_{ab}\right]=\begin{bmatrix} 0 & \cos\beta & 0 \\ -\cos\beta & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the matrix of direction cosines used for a strictly vertical finite element (Saouma 2000) during the first calculation iteration; α, β and γ are the angles between local axis x' and global axes x, y and z respectively (Fig. 1).

Quasidiagonal matrix $\left[\bar{K}_\tau\right]$ is constructed based on the tangent stiffness matrices of individual finite elements. The tangent stiffness matrix for the whole structure is expressed as:

$$\left[K_\tau\right]=\left[H\right]^T\left[\bar{K}_\tau\right]\left[H\right], \tag{18}$$

where $\left[H\right]$ is the matrix of correspondence between the finite element displacements of the structure and the displacements of the whole structure. This matrix defines finite element displacements corresponding to the displacements of the whole structure.

The above matrix is incorporated into the equilibrium equations written in terms of increments:

$$\left[K_\tau\right]\Delta\mathbf{u}=\Delta\mathbf{F}, \tag{19}$$

where $\Delta\mathbf{F}$ and $\Delta\mathbf{u}$ are the vectors of all nodal load increments of the structure and global displacement increments.

The numerical realization of the tangent stiffness method is performed by using the load control method of Newton-Raphson described in detail in works of Karkauskas (2007) and Karkauskas and Popov (2009a).

5. Strength conditions

The main task in formulating the problem of analysis is associated with choosing strength conditions. For space structures, it is significant to use strength conditions including not only tension-compression strength but also the strength of the structural element bending about both axes. Strength condition for a complex stress-strain state is usually given in construction codes and specifications regulating the design of building structures (AISC 2005).

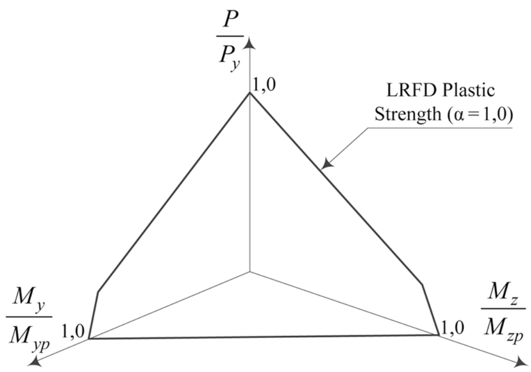

Fig. 2. The full plasticisation surface according to AISC-LRFD strength conditions (Kim *et al.* 2001)

According to construction specifications AISC-LRFD (Kim *et al.* 2001; Aminmansour 2000; AISC 2005), the double-axis plastic strength surface of the beam-column element (presented graphically in Fig. 2) is expressed by the equation given below:

$$1\geq\frac{P_j}{P_{y,j}}+\frac{8}{9}\frac{M_{y,j}}{M_{yp,j}}+\frac{8}{9}\frac{M_{z,j}}{M_{zp,j}}, \quad \textit{subject to}\ \frac{P_j}{P_{y,j}}\geq\frac{2}{9}\frac{M_{y,j}}{M_{yp,j}}+\frac{2}{9}\frac{M_{z,j}}{M_{zp,j}}, \tag{20.1}$$

$$1 \geq \frac{P_j}{2P_{y,j}} + \frac{M_{y,j}}{M_{yp,j}} + \frac{M_{z,j}}{M_{zp,j}}, \quad subject\, to \; \frac{P_j}{P_{y,j}} < \frac{2}{9}\frac{M_{y,j}}{M_{yp,j}} + \frac{2}{9}\frac{M_{z,j}}{M_{zp,j}}, \tag{20.2}$$

where P_j is longitudinal force applied to the element of the structure at the *j*-th cut; $P_{y,j}$ is limiting longitudinal force for the structural element under tension or compression at the *j*-th cut; $M_{y,j}$ is the bending moment of the "weak axis" of the cross-section at the *j*-th cut of the structural element; $M_{yp,j}$ is the limiting bending moment of the "weak axis" of the cross-section at the *j*-th cut of the structural element; $M_{z,j}$ is the bending moment of the "strong axis" of the cross-section at the *j*-th cut of the structural element; $M_{zp,j}$ is the ultimate bending moment of the "strong axis" of the cross-section at the *j*-th cut of the structural element.

All the above-mentioned limiting values of internal forces are obtained according to requirements for construction specification (AISC 2005). Local stability is achieved when compression members are loaded up to their critical forces. The values of the critical stresses of the compression member are obtained according to the codes. Once critical stresses are determined, critical forces are computed. The global stability of the whole construction can be checked in the following way. The determinant of the tangent stiffness matrix is obtained under condition of construction when strains are calculated. In case it is negative, there is loss of stability in all construction.

The problem of analysis may have two variants when strength conditions (20.1)–(20.2) are used. The first variant means that conditions (20.1)–(20.2) are used without any changes. This version of the problem can be written in terms of static (2.1)–(2.2) or kinematic formulation (5.1)–(5.3). The second variant implies reduction in strength conditions to one ultimate bending moment by multiplying both sides of conditions (20.1)–(20.2) by one of ultimate bending moments, for example, $M_{zp,j}$:

$$M_{zp,j} \geq c_{y,j}P_j + \frac{8}{9}m_{y,j}M_{y,j} + \frac{8}{9}M_{z,j}, \quad subject\, to \; c_{y,j}P_j \geq \frac{2}{9}m_{y,j}M_y + \frac{2}{9}M_{z,j}, \tag{21.1}$$

$$M_{zp} \geq \frac{1}{2}c_{y,j}P_j + m_{y,j}M_{y,j} + M_{z,j}, \quad subject\, to \; c_{y,j}P_j < \frac{2}{9}m_{y,j}M_y + \frac{2}{9}M_{z,j}, \tag{22.2}$$

where $c_{y,j} = M_{zp,j}/P_{y,j}$; $m_{y,j} = M_{zp,j}/M_{yp,j}$.

These expressions of AISC-LRFD strength conditions are given in terms of the linear function. The elastic-plastic analysis of the structure under such strength conditions can be performed using the formulations of problem (7.1)–(7.3), (8.1)–(8.3) or (11.1)–(11.2).

Though strength conditions are provided by construction codes and specifications, the efforts of various researchers were mad,e to accurately reflect a complex state of strains. An example is Orbison's full plastification surface of the cross-section (presented in Kim *et al.* (2001), Chiorean, Barsan (2005)) described as follows:

$$1 \geq 1{,}15p_j^2 + m_{z,j}^2 + m_{y,j}^4 + 3{,}67p_j^2m_{z,j}^2 + 3{,}0p_j^6m_{y,j}^2 + 4{,}65m_{z,j}^4m_{y,j}^2, \tag{23}$$

where: $p_j = P_j/P_{y,j}$; $m_{z,j} = M_{z,j}/M_{pz,j}$ (a "strong" element cross-section axis (Fig. 3)); $m_{y,j} = M_{y,j}/M_{py,j}$ (a "weak" element cross-section axis (Fig. 3)).

Orbison's plastic surface is shown in Fig. 3.

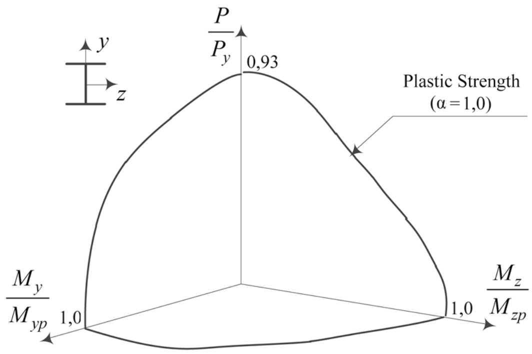

Fig. 3. Full plastification surface according to Orbison's strength conditions (Kim *et al.* 2001)

All above-mentioned conditions do not take into consideration the effect of torsional strains on the complex state of strains. Consequently, to get comprehensive information about strength, some torsional strength conditions should be used. Torsional strength condition given by Eurocode 3 (2006) is expressed by the equation:

$$\frac{T_{Ed}}{T_{Rd}} \leq 1{,}0, \tag{24}$$

where: T_{Ed} is torsion moment applied to the structural element; T_{Rd} is torsion strength specified for the structural element.

The torsion strength of the cross-section specified in works by Ikrin (2005) and Trahair *et al.* (2008) is estimated by the formula:

$$T_{Rd} = \tau_y W_t, \tag{25}$$

where: τ_y is yield shear strain; W_t is the modulus of torsion evaluated for the open-end thin-walled cross-section in the following way:

$$W_t = \frac{I_t}{t_{\max}}, \tag{26}$$

where: I_t is torsion constant; $t_{\max}$ is the maximum thickness of the considered cross-section.

6. Numerical realization

MATLAB mathematical modelling software was chosen for the numerical realization of the problems faced in the analysis. This program along with the Optimization Toolbox allows us to solve various problems of complex optimization. To deal with a problem of analyzing geometrically nonlinear elastic-plastic structure expressed employing mathematical model (7.1)–(7.3) when strength conditions are given in linear dependences, optimization tool "quadprog", which can solve the problems of quadratic programming, should be used. The following equation is used for solving the considered problem:

find

$$\min_{x} \quad \frac{1}{2}\mathbf{x}^T[H]\mathbf{x}+\mathbf{f}^T\mathbf{x} \tag{27.1}$$

subject to

$$\begin{cases}[A]\mathbf{x}\le\mathbf{b},\\ [A_{eq}]\mathbf{x}=\mathbf{b}_{eq},\\ \mathbf{lb}\le\mathbf{x}\le\mathbf{ub},\end{cases} \tag{27.2}$$

where: $\mathbf{x}$ is the vector of variables dealing with the optimization problem; $[H]$ is a symmetric matrix of coefficients of quadratic terms in the quadratic equation; $\mathbf{f}$ is the vector of coefficients of linear terms in the quadratic equation; $[A]$ is the matrix of coefficients of linear inequality constraints; $\mathbf{b}$ is the vector of absolute terms of linear inequality constraints; $[A_{eq}]$ is the matrix of coefficients of linear equality constraints; $\mathbf{b}_{eq}$ is the vector of absolute terms of linear equality constraints; $\mathbf{lb}$ is the vector of the lower bounds of variables dealing with the optimization problem; $\mathbf{ub}$ is the vector of the upper bounds of variables dealing with the optimization problem.

The problem of analyzing geometrically nonlinear elastic-plastic structure expressed by mathematical model (2.1)–(2.3) when strength conditions are expressed in terms of nonlinear dependences should be solved by optimization tool "fmincon" that finds the minimum of the constrained nonlinear multivariable function. The equation for solving the problem is as follows:

find

$$\min_{x}\mathbf{f}(\mathbf{x}), \tag{28.1}$$

subject to

$$\begin{cases}[c(\mathbf{x})]\le\mathbf{0},\\ [c_{eq}(\mathbf{x})]=\mathbf{0},\\ [A]\mathbf{x}\le\mathbf{b},\\ [A_{eq}]\mathbf{x}=\mathbf{b}_{eq},\\ \mathbf{lb}\le\mathbf{x}\le\mathbf{ub},\end{cases} \tag{28.2}$$

where: $\mathbf{x}$ is the vector of variables dealing with the optimization problem; $f(\mathbf{x})$ is the objective function of the optimization problem; $[c(\mathbf{x})]$ is the matrix of nonlinear inequality constraints expressed by functions; $[c_{eq}(\mathbf{x})]$ is the matrix of nonlinear equality constraints expressed by functions; $[A]$ is the matrix of coefficients of linear inequality constraints; $\mathbf{b}$ is the vector of absolute terms of linear inequality constraints; $[A_{eq}]$ is the matrix of coefficients of linear equality constraints; $\mathbf{b}_{eq}$ is the vector of absolute terms of linear equality constraints; $\mathbf{lb}$ is the vector of the lower bounds of variables dealing with the optimization problem; $\mathbf{ub}$ is the vector of the upper bounds of variables dealing with the optimization problem.

It should be mentioned it is sufficient to use MATLAB to solve the problem of direct optimization, and thus obtain variables helping with solving the problem of dual formulation.

7. Two-storied space structure

A two-storied space frame structure was chosen for the numerical realization of the above-mentioned mathematical models. This structure was studied by Kim *et al.* (2003) (testing the full-size model made of real steel profiles was described) and Kim and Lee (2002) (the analysis of a discrete computer-generated model in the environment of program ABAQUS was presented). In the latter research, the obtained results were compared with data obtained in the former research.

Structural elements are modelled using the profiles of type H150×150×7×10. The overall dimensions of the frame (Fig. 4) are as follows: width in the direction of axis x is 2.5 m, length in the direction of axis y – 3.0 m, height from the column element base to the second floor

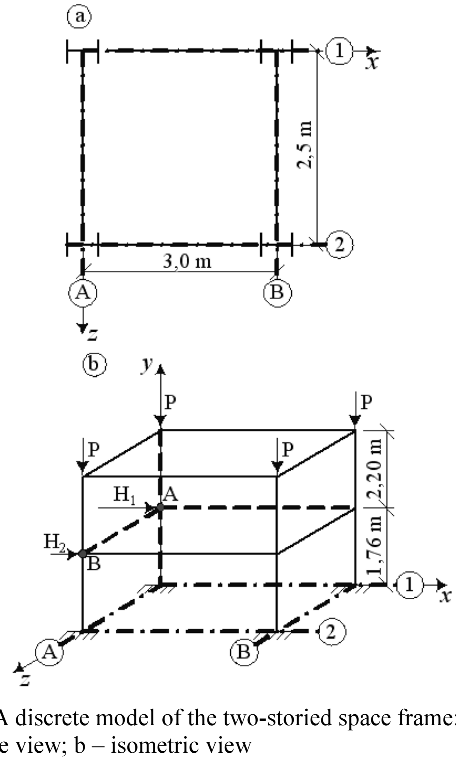

Fig. 4. A discrete model of the two-storied space frame: a – plane view; b – isometric view

level – 1,76 m and height from the second floor level to the roof is 2,2 m. The geometrical and physical parameters of the cross-section of any structural element are constant through its full length. The yield strength of all structural elements – 320 MPa while the elastic modulus is 221 GPa and the shear modulus is 85 GPa. The model of the structure is subjected to the action of three various types of loads presented in Table 1.

Two calculations were performed for every load case. The first calculation was made applying Orbison's strength conditions (23) and the second – AISC-LRFD standard strength conditions (20.1)–(20.2) or (21.1)–21.2). The results obtained in the performed two-storied space frame analysis were compared with data presented in work by Kim and Lee (2002).

Table 1. Load cases of the analysed frame

Load case	Vertical load	Horizontal load (H_1)	Horizontal load (H_2)
1	P	P/5	P/10
2	P	P/4	P/8
3	P	P/3	P/6

Fig. 5 shows the sequence of plastic hinge formation and the shape of a deformed frame under ultimate load. Plastic hinges initially occur in the columns along axis 1 near supports. With an increase in loads, plastic hinges spread over the tops of these columns, whereas later – over the columns along axis 2. Since horizontal loads are asymmetrical, torsional forces are induced and the first floor columns of the frame deform in a twisting mode. The sequence of plastic hinge formation corresponding to the load ratios is shown in Table 2 considering various strength conditions when analyzing the problem. The data were obtained in the first case of the load choosing value P equal to 675 kN. Load-displacement curves along global axis x for nodes A and B and all load cases are shown in Figs 6–11.

The carried out analysis revealed that under Orbison's strength conditions (23), the results closely approached data obtained by Kim and Lee (2002). A greater deviation from the results presented in this work could be observed when strength conditions specified by AISC-LRFD construction code (20.1)–(20.2) or (21.1)–(21.2)

Table 2. A comparison of plastic hinge load ratios for a two-storied space frame

Sequence of hinge formation	Load ratio (AISC-LRFD)	Load ratio (Orbison)
1	0,423	0,740
2	0,432	0,754
3	0,514	0,882
4	0,530	0,889
5	0,543	0,918
6	0,562	0,943
7	0,666	0,945
8	0,681	0,965

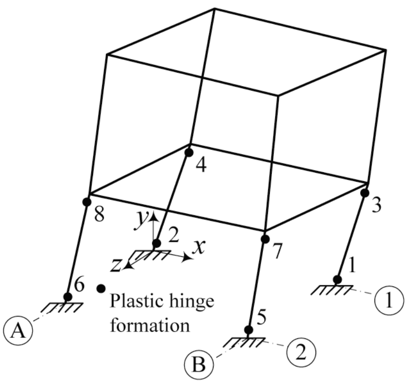

Fig. 5. The shape of a deformed two-storied space frame under ultimate load for the first case of the load

were used. These conditions are more strict or conservative than those of Orbison's strength. It can be explained by the fact that AISC-LRFD is the construction code regulating structural steel design. While applying such standards, it is usually assumed that the structure works only within elastic work limits. The characters of load-displacement diagrams and its comparison allow making a conclusion that construction work is obtained by the proposed algorithm and agree well with Kim *et al.* (2003) results.

The results of calculating the values of limit forces are presented in Table 3. When performed calculations suggest the underestimated load carrying capacity of the frame applying Orbison's strength criterion in comparison with the results provided by Kim and Lee (2002), the difference makes from 13.5% to 21.5%. The produced error employing the linear yield surface of the AISC-LRFD code is from 36.8% to 41.1%. Such differences can be caused by accurately measured geometrical imperfections in the profile and physical parameters and are presented in works by Kim *et al.* (2003). For example, yield limits obtained for vertical elements were 320 MPa for flanges and 311 MPa for the web. For horizontal elements, yield limits made 344 MPa for flanges and 327 MPa for the web. In this work, the calculation model is more idealized. There are no changes in the geometry of the profile cross-section or physical parameters in light of element length.

Table 3. The values of limit forces at nodes "A" and "B"

Nodes	(Kim *et al.* 2002) results		Proposed algorithm results	
	Full-size test results	ABAQUS calculation results	Orbison strength conditions	AISC-LRFD code strength conditions
I load type				
"A"	151,8 kN	151,5 kN	130,0 kN	95,0 kN
"B"	75,1 kN	75,7 kN	65,0 kN	47,5 kN
II load type				
"A"	169,5 kN	171,9 kN	144,5 kN	106,8 kN
"B"	84,7 kN	85,8 kN	73,0 kN	53,4 kN
III load type				
"A"	204,0 kN	198,1 kN	160,0 kN	120,0 kN
"B"	99,1 kN	101,9 kN	80,0 kN	60,0 kN

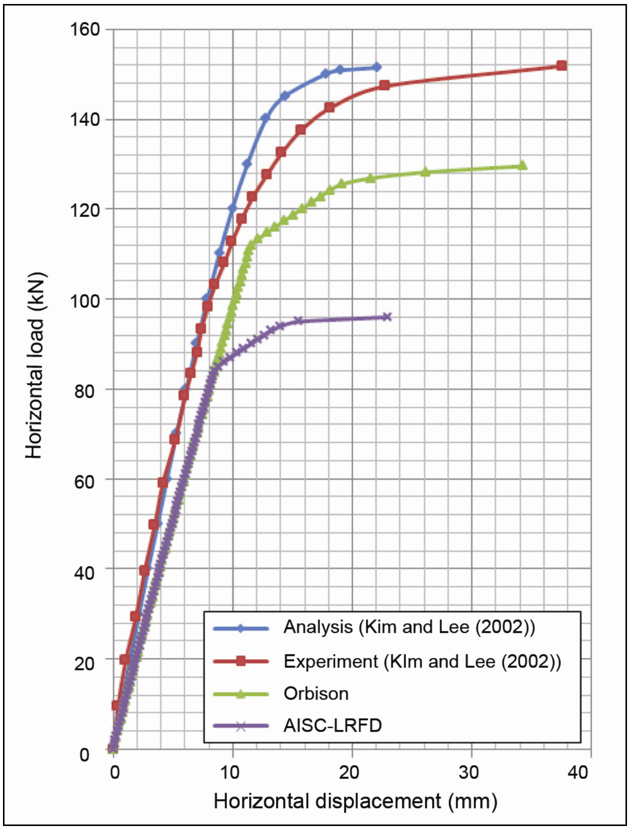

Fig. 6. A load-displacement curve of node A in the first case of the load

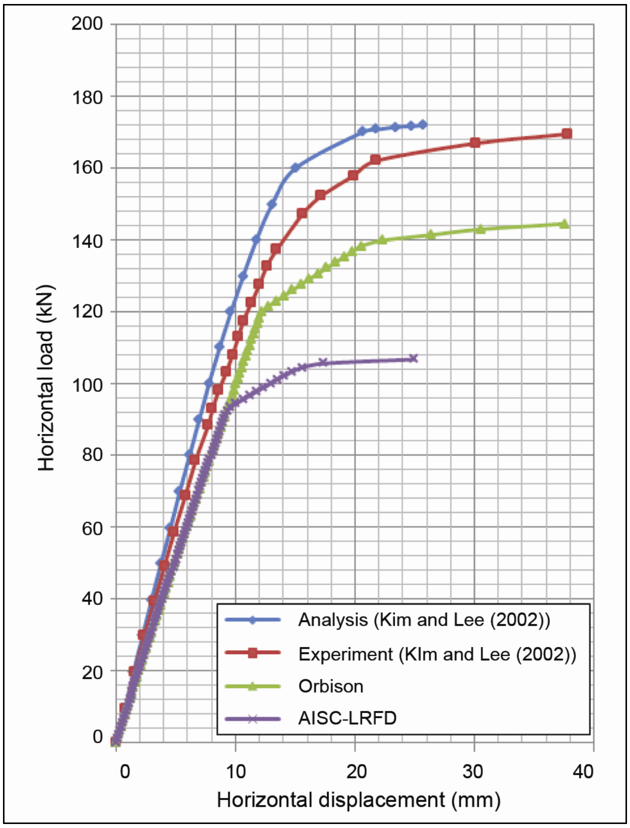

Fig. 8. A load-displacement curve of node A in the second case

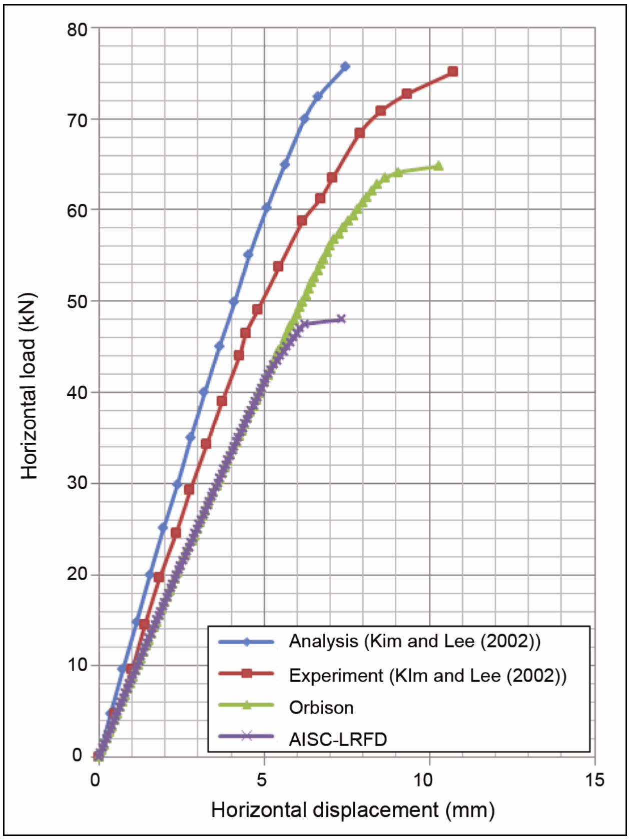

Fig. 7. A load-displacement curve of node B in the first case of the load

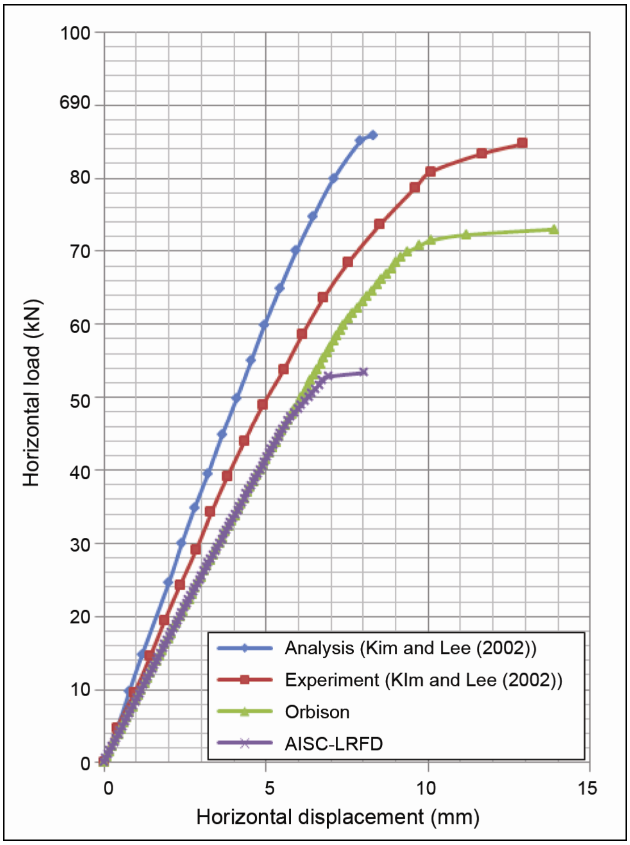

Fig. 9. A load-displacement curve of node B in the second case

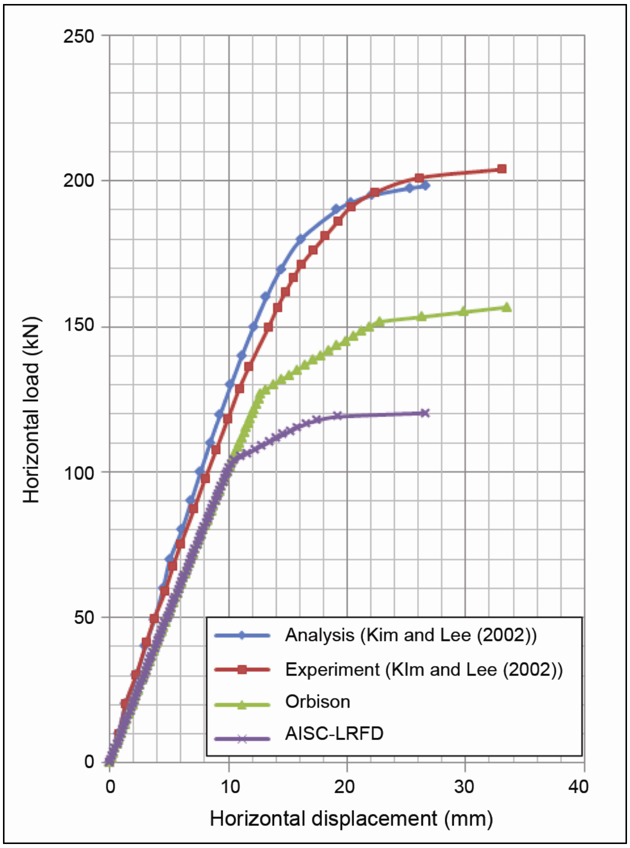

Fig. 10. A load-displacement curve of node A in the third case

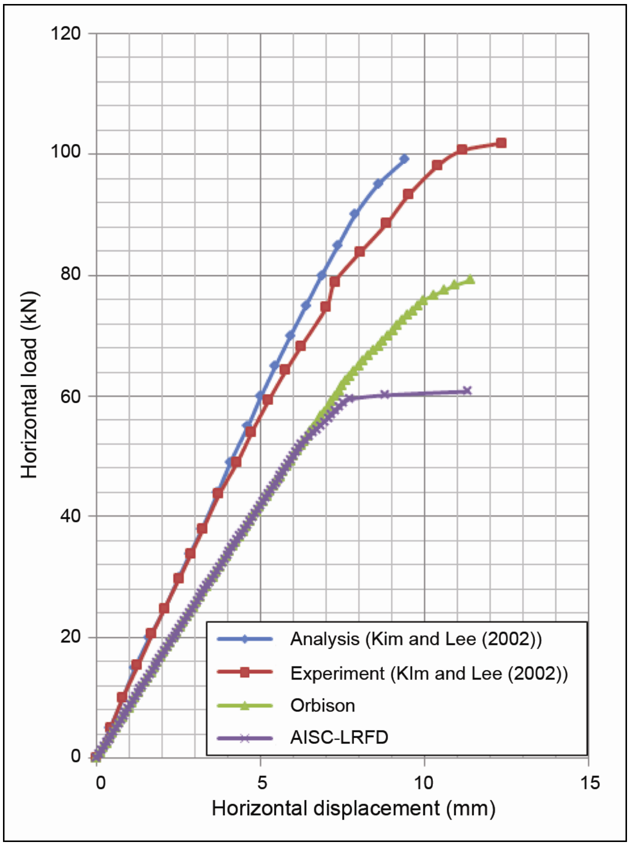

Fig. 11. A load-displacement curve of node B in the third case

8. Conclusions

Mathematical models and a technique for constructing a tangent stiffness matrix for space finite elements were developed for analyzing the dual formulation problem of geometrically nonlinear elastic-plastic space structure. The observed different strength conditions for a complex stress-strain state were used in the development of mathematical models for analyzing the considered problem.

Numerical realization was performed to obtain the real stress-strain state of a two-storied space frame. The validity of the formulation of the analysed problem was checked by comparing the obtained results with data presented in literature.

In further research, mathematical models developed for the analysis of the above discussed problem will be used for considering optimization problems with complex constraints.

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TAMPRIAI PLASTINIŲ GEOMETRIŠKAI NETIESINIŲ ERDVINIŲ RĖMŲ ANALIZĖ

R. Karkauskas, M. Popov

Santrauka

Projektuojant ar rekonstruojant konstrukcijas, atliekant jos elementų skerspjūvių optimizavimo skaičiavimus, vienas iš svarbiausių uždavinių – konstrukcijos tikrojo įtempto deformuoto būvio (ĮDB) nustatymas. Tai galima pasiekti atliekant konstrukcijos kaip geometriškai netiesinės sistemos (atsisakant mažų poslinkių prielaidos) analizę, įvertinant plastines deformacijas. Taikant šiuolaikines kompiuterines technologijas ir matematinį aparatą, tapo įmanoma vykdyti erdvinės konstrukcijos stiprumo analizę ir padidinti konstrukcijos ĮDB analizės tikslumą. Tuo tikslu šiame darbe toliau plėtojama tangentinės standumo matricos sudarymo metodika erdvinės rėminės konstrukcijos netiesinei analizei, įvertinant plastines deformacijas. Naudojant matematinio programavimo dualumo teoriją sudaryti analizės statinės ir kinematinės formuluočių uždavinių matematiniai modeliai. Naudojamos AISC-LRFD normatyviniuose dokumentuose pateiktos ir kitų autorių (pavyzdžiui, Orbison) pasiūlytos stiprumo sąlygos. Suformuluoti analizės uždavinių matematiniai modeliai buvo aprobuoti skaičiuojant dviejų aukštų erdvinį rėmą. Gauti analizės rezultatai palyginti su eksperimentiniais ir kitų autorių analitiniais rezultatais.

Reikšminiai žodžiai: tampriai plastinė erdvinė konstrukcija, geometrinis netiesiškumas, tangentinė standumo matrica, antrosios eilės analizė, MatLAB.

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