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Investigation of Stationary Solutions of Viscoelastic Melt Spinning Equations and Stability with Respect to Increasing Viscoelasticity

R. Dhadwal and S.K. Kudtarkar

Center for Mathematical Modeling, FLAME
150/7,"Jaideep Bungalow", Prathamesh Park, Baner-Balewadi Road,
Baner, Pune, India
E-mail(*corresp.*): renu.dhadwal29@gmail.com;renu@flameedu.in

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Abstract. The one-dimensional equations governing the formation of viscoelastic fibers using Giesekus constitutive equation were studied. Existence and uniqueness of stationary solutions was shown and relation between the stress at the spinneret and the take-up velocity was found. Further, the value of the Giesekus model parameter for which the fibre exhibits Newtonian behaviour was found analytically. Using numerical simulations it was shown that below this value of the parameter the fluid shows extension thickening behaviour and above, extension thinning. In this context, by simulating the non-stationary equations the effect of viscoelasticity on the stability of the spinning process was studied.

Keywords: melt spinning, Giesekus model parameter, viscoelasticity, analytic stationary solutions.

AMS Subject Classification: 34B15; 76A10.

1 Introduction

Melt spinning is the industrial process of manufacturing long, slender fibres. In this process, molten polymer is extruded from a pressurized reservoir through a small circular orifice called the spinneret. The liquid jet undergoes stretching, cooling and solidification. The solidified filament is then wound up via some take-up device at a velocity much higher than the extrusion velocity to ensure that the fibre is stretched. The ratio of the take-up velocity (v_L) to the extrusion velocity (v_0) is called the draw ratio ($D = v_L/v_0 > 1$). The draw ratio is a crucial factor in determining the stability of the spinning process. The spinning process is said to be unstable when, perturbing the take-up velocity slightly results in big perturbations in the cross-sectional area of the fibre. This is observed to happen when the draw ratio exceeds a critical value. This instability is called the draw resonance and is characterized by sustained periodic oscillations in the cross-sectional area of the fibre.

The dynamics of melt spinning has been studied extensively in the past few decades by several research groups, see [1, 3, 11, 18]. In recent times, more and more sophisticated models have been developed which take into account microstructure, crystallization and viscoelastic effects in the simulation of the spinning process, [2, 4, 5]. There have been many attempts with regard to understanding the draw resonance mechanism, see [6, 9, 10]. Stability of the spinning process has been investigated using linear or spectral stability analysis by several research groups and the effect of various parameters on the stability has been investigated, as in [7, 9, 10, 11, 15, 16]. However, it has been an uphill task to investigate the global solvability of these equations. Mechanics of steady state spinning was investigated first by Matovich and Pearson [14] for viscous fluid in elongational flow. In recent times Hagen and Renardy [8] have studied forced elongation of viscous fluids and proved the existence of non-stationary solutions using semigroup theory. Hagen has also proved the existence of non-stationary solutions of non-isothermal viscoelastic melt spinning equations in which the main assumption is that the temperature of the fibre is monotonically decreasing and reaches a certain value (called the solidification point). This assumption is crucial to all the results proved in that paper.

In this study the isothermal case has been considered and the purpose of this paper is two-fold. First, we investigate stationary solutions of isothermal melt spinning equations using the Giesekus constitutive equation. The Giesekus constitutive equation is employed because this model describes both extensional thickening and extensional thinning behaviour and is employed widely in the simulation of fibre spinning process. From the analytic solution, the value of the Giesekus model parameter is got for which the fluid shows Newtonian behaviour (constant viscosity) and which also serves as the borderline between extensional thickening and extensional thinning behaviour. It may be added that in a similar way existence and uniqueness results can be proved for the spinning equations with any other viscoelastic constitutive equation.

Second, the study of stability of the spinning process translates mathematically into the study of the stability of the equilibrium solutions. Therefore, equipped with the existence results of the stationary solutions, stability of these stationary solutions with respect to changing viscoelasticity is investigated. This is done by simulating the complete system of nonstationary melt spinning equations with perturbed boundary conditions for different values of the Deborah number (De). The effect of viscoelasticity on the spinning process has been studied previously by Lee et al, [13]. White Metzner and PTT fluids were used in their studies. It was reported by them that for a certain value of model parameter, the fluid changed its behaviour from extension thickening to extension thinning. The effect of viscoelasticity on stability of the fibre depended on whether the fluid was extension thickening or extension thinning. For extension thickening fluids increase in viscoelasticity increased stability whereas for extension thinning fluids it decreased the stability. In our study these results are confirmed and furthermore effect of model parameter on the stability is also investigated.

2 Description of the System

A polymer fibre is modelled as a uniaxial, extensional flow of a viscoelastic fluid. Considering the geometry of the fibre, usually a cylindrical coordinate system is used to describe the flow with the z coordinate in the direction of the flow. When the polymer melt exits the spinneret, it swells to several times its diameter size. This is a characteristic behaviour of non-newtonian fluids, [17]. While modelling the fibre mathematically however, this "die-swell" phenomenon has been neglected. Considering the conservation laws of mass and momentum of a polymer jet and averaging over the cross-section of the fibre, the following one-dimensional (1-d) equations are got: [12].

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial z}(Av_z) = 0, \tag{2.1}$$

$$\rho A \left(\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} \right) = \frac{\partial}{\partial z}(A\tau_{zz}). \tag{2.2}$$

A detailed derivation of the 1-d spinning equations done by integrating the equations of conservation of mass and momentum over the cross sectional area of the fibre can be found in [12]. In the above equations, z denotes the coordinate along the spinline, t is the time, A the cross sectional area of the fibre, v_z the axial velocity and τ_{zz} the axial stress. In the momentum equation, force due to gravity, surface tension and force due to air drag have been neglected for simplicity. Later, the force due to inertia will also be neglected. Radial stress variable is not considered. The energy equation has been neglected in order to get the isothermal process.

The constitutive equation of the Giesekus fluid has the form, [17]:

$$\boldsymbol{\tau} + \lambda \left(\frac{D\boldsymbol{\tau}}{Dt} - \mathbf{D}.\boldsymbol{\tau} - \boldsymbol{\tau}.\mathbf{D}^T \right) + \alpha \boldsymbol{\tau}.\boldsymbol{\tau} = 2\mu\dot{\epsilon},$$

where $\boldsymbol{\tau}$ denotes the stress tensor, $\mathbf{D}$ represents the deformation rate tensor, λ is the relaxation time of the polymer (time taken for the fluid to get back its original state after being stretched), μ is the zero-shear viscosity, α is a material constant and $\dot{\epsilon} = (\mathbf{D} + \mathbf{D}^T)/2$ denotes the rate-of-strain or extensional rate tensor. In the above equation $\frac{D}{Dt}$ denotes the material derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}.\mathit{\Delta}.$$

For simplicity, considering only the axial stress variable τ_{zz}, and integrating over the cross-sectional area, we get the following form of the Giesekus constitutive equation

$$\tau_{zz} + \lambda \left(\frac{\partial \tau_{zz}}{\partial t} + v_z \frac{d\tau_{zz}}{dz} - 2\tau_{zz}\frac{dv_z}{dz} \right) + \alpha\tau_{zz}^2 = 2\mu\frac{dv_z}{dz}. \tag{2.3}$$

The system (2.1), (2.2) and (2.3) is subject to the following boundary conditions

$$A = A_0, \quad v_z = v_0, \quad \text{at } z = 0 \quad \text{for all} \quad t; \quad v_z = v_L \quad \text{at } z = L \quad \text{for all } t,$$

where L denotes the length of the fibre, A_0 and v_0 denote the cross sectional area and axial velocity of the fibre at the spinneret and v_L denotes the take-up velocity of the fibre.

2.1 Dimensionless form

Introducing the dimensionless quantities,

$$a^* = \frac{A}{A_0}, \quad v^* = \frac{v_z}{v_0}, \quad t^* = \frac{tv_0}{L}, \quad \tau^* = \frac{\tau_{zz}L}{2\mu v_0}, \quad z^* = \frac{z}{L}, \quad De = \frac{\lambda v_0}{L}$$

and dropping the star, the following dimensionless transport equations governing the melt spinning process along with the Giesekus constitutive equation are obtained

$$\frac{\partial a}{\partial t} + \frac{\partial (av)}{\partial z} = 0, \tag{2.4}$$

$$\frac{d(a\tau)}{dz} = 0, \tag{2.5}$$

$$\frac{\partial \tau}{\partial t} + v\frac{\partial \tau}{\partial z} - \big(2\tau + 1/De\big)\frac{\partial v}{\partial z} = -\frac{\tau}{De}\big(1 + 2\alpha De\tau\big), \tag{2.6}$$

$$a = 1, \quad v = 1, \quad z = 0, \tag{2.7}$$

$$v = D > 1, \quad z = 1, \tag{2.8}$$

where $D = v_L/v_0$ is the draw ratio. The parameter α is such that $0 < \alpha \leq 1$. In the momentum equation (2.5), the inertia term has been dropped for simplicity. In Eq.(2.6), De denotes the Deborah number which is the ratio of the relaxation time of the fluid to the characteristic time scale of the flow. For $De \ll 1$, the fluid relaxes relatively quickly and behaves like a viscous fluid, and for $De \gg 1$, the fluid does not relax on the time scale of the flow, and therefore behaves more like an elastic solid, [17].

3 Existence of Stationary Solutions

From the system (2.4)–(2.6), we can easily read off the stationary system of melt spinning equations.

$$\frac{d(av)}{dz} = 0, \quad \frac{d(a\tau)}{dz} = 0, \tag{3.1}$$

$$v\frac{d\tau}{dz} - \left(2\tau + \frac{1}{De}\right)\frac{dv}{dz} = -\frac{\tau}{De}(1 + 2\alpha De\tau) \tag{3.2}$$

The boundary conditions are given by (2.7) and (2.8).

Proposition. *Under the assumption that* $v(z) > 0 \ \ \forall z \in [0,1], De > 0$ *and* $\tau_0 > 0$*, there exists a unique solution for equations* (3.1)–(3.2) *along with* (2.7) *and* (2.8) *such that* v *is monotonically increasing.*

Proof. From equations (3.1) we readily obtain

$$av = 1, \quad a\tau = \tau_0(D), \tag{3.3}$$

where τ_0 is the value of stress at the spinneret. This value is unknown and dependent on the draw ratio D. From the above equations we get $\tau = \tau_0(D)v$. Substituting this into (3.2), we get the following differential equation in variable $v(z)$.

$$\frac{dv}{dz}(De\, v\tau_0(D) + 1) = v\tau_0(D)(1 + 2\alpha De\, v\tau_0(D)). \tag{3.4}$$

Solving the above differential equation and applying the boundary condition $v(0) = 1$, we get the solution in the following form

$$v\left(\frac{1 + 2\alpha Dev\tau_0(D)}{1 + 2\alpha De\tau_0(D)}\right)^{\frac{1-2\alpha}{2\alpha}} = \exp(z\tau_0(D)). \tag{3.5}$$

Let

$$g(v) = v\left(\frac{1 + 2\alpha Dev\tau_0(D)}{1 + 2\alpha De\tau_0(D)}\right)^{\frac{1-2\alpha}{2\alpha}} - \exp(z\tau_0(D)), \quad \forall z \in (0,1).$$

We observe that,

$$\lim_{v\to 0} g(v) = -\exp(\tau_0(D)z) < 0,$$
$$g(v) \to \infty \ \ as \ \ v \to \infty.$$

This shows that the function g has at least one zero in the domain $[0,\infty)$. But before we can claim the existence of the solution to the spinning equations we need to prove that given D, there exists τ_0 such that $v(1) = D$. For this we first find the relationship between τ_0 and D which is easily obtained by substituting the boundary condition $v(1) = D$ in (3.5):

$$D\left(\frac{1 + 2\alpha DeD\tau_0}{1 + 2\alpha De\tau_0}\right)^{\frac{1-2\alpha}{2\alpha}} = \exp(\tau_0). \tag{3.6}$$

Therefore, finding τ_0 is equivalent to the problem of finding a zero of the function f, where

$$f(\tau_0) = D\left(\frac{1 + 2\alpha DeD\tau_0}{1 + 2\alpha De\tau_0}\right)^{\frac{1-2\alpha}{2\alpha}} - \exp(\tau_0).$$

We readily obtain that

$$\lim_{\tau_0\to 0} f(\tau_0) = D - 1 > 0 \quad \text{for} \quad D > 1,$$
$$f(\tau_0) \to -\infty \ \ \text{as} \ \ \tau_0 \to \infty.$$

Again, applying the intermediate value theorem, f has at least one zero in $[0\ \infty)$. Therefore, we can find $\tau_0 > 0$ such that there exists at least one solution $v(z)$ which fulfils the boundary condition $v(1) = D$.

To prove the uniqueness of the solution it is enough to prove that $g(v) \neq 0$ for $v \in (0, \infty)$ for all $z \in [0, 1]$. First, we note that g is continuous and differentiable for $v \in (0, \infty)$.

$$\frac{dg}{dv} = \left(\frac{1 + 2\alpha De\tau_0 v}{2\alpha De\tau_0}\right)^{\beta} \left(1 + \frac{v\beta 2\alpha De\tau_0}{1 + 2\alpha De\tau_0 v}\right),$$

where $\beta = \frac{1-2\alpha}{2\alpha}$. It can be easily seen that for $v > 0$, $De > 0$, $\tau_0 > 0$, $\frac{dg}{dv} \neq 0$. This proves that there can exist only one v such that $g(v) = 0$. From Eq.(3.4) it is easy to see that $\frac{dv}{dz} > 0 \quad \forall z \in [0, 1]$. This implies that v is monotonically increasing in the fibre domain. □

Equation (3.6) is an important one because it gives the relation between the stress at the spinneret and the draw ratio. Usually, the boundary conditions prescribed are the diameter and velocity at the spinneret and the take-up velocity at the end of the fibre. The boundary value problem is then solved using a shooting method, where the stress at the spinneret has to be guessed. A relation such as (3.6) eliminates the need for a shooting method hence improves the efficiency of numerics.

3.1 Qualitative behaviour of Giesekus fluid as a function of the parameter α

The motivation of this section is based on the studies done by Lee et al [13], on the effect of fluid viscoelasticity on stability of the spinning process. They conducted their studies using White-Metzner and PTT fluids. According to their studies the effect of increasing viscoelasticity depended on whether the fluid was extension thickening or extension thinning. This in turn depends on the parameters of the viscoelastic models. They found numerically the value of the parameter below which the fluid showed extension thickening behaviour and above which the fluid showed extension thinning behaviour.

In our work we find analytically that value of the Giesekus model parameter α for which the fluid becomes Newtonian (with constant viscosity). This value of the parameter serves as the borderline between extension thickening and extension thinning fluids.

Let η denote the extensional viscosity of the fluid and let $\dot{\varepsilon} = dv/dz$ denote the extension rate of the fibre. Then,

$$\eta(\dot{\varepsilon}) = \tau/\dot{\varepsilon}. \tag{3.7}$$

For uniaxial extensional flow of Non-newtonian fluids, the extensional viscosity depends on the extension rate. If this viscosity increases with increasing elongation rate the fluid is known as extension thickening fluid and if it decreases with increasing extension rate the fluid is called extension thinning. Here, we derive the solution when the viscosity is constant. For such fluid, the relation between the stress and the extension rate is linear. Hence, $\tau = \eta\dot{\varepsilon} = K\dfrac{dv}{dz}$ where η is constant. But from Eqns. (3.3), we also know that $\tau = \tau_0 v$. Therefore,

$$\tau = \eta\frac{dv}{dz} \iff \tau_0 v = \eta\frac{dv}{dz}.$$

From Eq.(3.4), we get

$$\eta = \frac{1 + Dev\tau_0}{1 + 2\alpha Dev\tau_0}. \tag{3.8}$$

Our claim that η is constant is true if and only if $\alpha = 0.5$. The solution for $\alpha = 0.5$ is given by

$$v(z) = D^z. \tag{3.9}$$

Therefore, the Giesekus fluid shows Newtonian behaviour for $\alpha = 0.5$. This is supported by numerical evidence as shown in Fig. 1, where the extensional viscosities at various spinline positions have been plotted against the extension rates at the same positions.

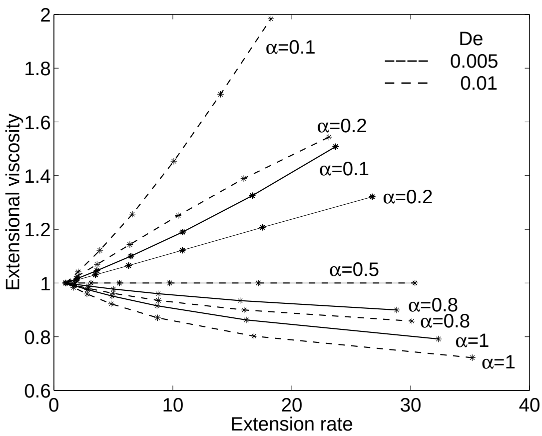

Figure 1. Extensional viscosity versus Extension rate.

The extensional viscosities have been calculated by simulating the stationary equations (3.1)–(3.2) and (2.7)–(2.8). From the figure, we see that for $\alpha = 0.5$, the extensional viscosity is constant with respect to the extensional rate showing Newtonian behaviour. For $\alpha < 0.5$, the extensional viscosity increases with increasing extension rate hence showing that the fluid exhibits extension thickening behaviour. For $\alpha > 0.5$, the extensional viscosity is seen to be decreasing with increasing extension rate thus showing that the fluid exhibits extension thinning behaviour.

4 Non-Stationary Equations

In this section, equipped with the existence results of the stationary solutions we study the stability of these stationary solutions by simulating the nonstationary equations as given by Eqs. (2.4)–(2.6) along with the following boundary conditions.

$$\begin{aligned} t = 0: &\quad a = a_s, \quad v = v_s, \quad \tau = \tau_s \quad \text{for} \quad 0 < \mathrm{z} < 1, \\ t > 0: &\quad a = a_0 = 1, \quad v = v_0 = 1, \quad \text{at} \quad z = 0, \\ t > 0: &\quad v = D(1+\epsilon) \quad \text{at} \quad z = 1. \end{aligned}$$

Here v_s, a_s and τ_s represent the steady state profiles of velocity, cross sectional area and stress respectively. The disturbance in the draw ratio is represented by $\epsilon < 1$.

4.1 Note on the numerical method

The numerical method used to solve the system of equations is the method of lines. We first discretise the equations in time. This results in the reduction of the partial differential equation (pde) system to an ordinary differential equation (ode) system with spinlength as the independent variable. We set $h = 1/N$, $t_n = n/N$, $0 \le n \le N$ and define

$$a_n(z) = a(t_n, z), \;\; v_n(z) = v(t_n, z), \;\; \tau_n(z) = \tau(t_n, z), \;\; 0 \le n \le N.$$

Let $u_n(z) = [a_n(z)\ v_n(z)\ \tau_n(z)]'$. The discretization of Eqns. (2.4)–(2.6) in the time variable gives

$$\frac{du_n}{dz} = B_n^{-1}\left(f_n - A\left(\frac{u_n - u_{n-1}}{h}\right)\right) \quad 1 \le n \le N, \tag{4.1}$$

where

$$B_n = \begin{pmatrix} v_n & a_n & 0 \\ \tau_n & 0 & a_n \\ 0 & -(2\tau_n + 1/De) & v_n \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$f_n = \begin{pmatrix} 0 \\ 0 \\ -\tau_n(1/De + 2\alpha\tau_n) \end{pmatrix}$$

In addition we impose the initial condition $u_0(z) = u_s$, where u_s is the steady state solution, and the boundary conditions

$$a(n,0) = 1, \quad v(n,0) = 1, \quad v(n,1) = D(1+\epsilon).$$

This system of ordinary differential equations is treated using standard differential algebraic solvers. At each time step the shooting method is used to match the axial velocity at the fibre end with the prescribed final velocity in order to solve the boundary value problem. It is worthwhile to note that although we found the relation between the stress at the spinneret and the take up velocity for the stationary equations, for the non-stationary equations this stress would be dependent on time too and hence the need for shooting method at each time step. The steady state solutions are got by solving the stationary equations (3.1)-(3.2) with the boundary conditions (2.7) and (2.8) using the MATLAB ode solver ode23tb. The convergence of both the stationary and non-stationary simulations depends crucially on the guessed value of τ_0. In the stationary case, this is done by the relation (3.6). In the non-stationary case, at every time step two values of τ_0 are guessed and the system (4.1) is solved. Then a secant method is used to get a new value of τ_0. This is continued till the value of τ_0 converges to a prescribed tolerance. The transient solutions were computed for 1000 time steps.

4.2 Results of numerical simulation and discussion

The results of the numerical simulations of the non-stationary equations can be seen in Fig. 2, where the cross-sectional areas and the the spinline tensions of the fibre at the take-up position ($z = 1$) have been plotted against time for two different values of α representing extension thickening ($\alpha = 0.2$) and extension thinning ($\alpha = 0.8$) fluids.

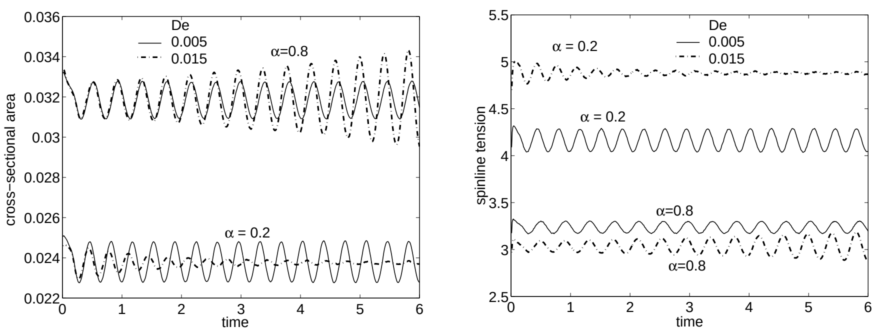

Figure 2. Transient response of the cross sectional area (left) and spinline tension (right) at take-up for extension thickening fluids ($\alpha = 0.2, D = 40$) and extension thinning fluids ($\alpha = 0.8, D = 30$).

We observe that consistent with results reported by Lee [13], increase in viscoelasticity increases stability for extension thickening fluids and it decreases stability for extension thinning fluids. In their work they explained that stability depended on the spinline tension sensitivity. Higher the tension, lower is the tension sensitivity and vice versa. This tension sensitivity in turn is responsible for the cross sectional area sensitivity. It is mainly the latter which determines the stability of the spinning process. In order to understand how the increase in viscoelasticity can increase or decrease the spinline tension we again take a look at Eq. (3.8).

$$\eta = (1 + Dev\tau_0)/(1 + 2\alpha Dev\tau_0).$$

From the above it can easily be seen that η increases as De increases for $\alpha < \frac{1}{2}$ (extension thickening fluids) and that η decreases as De increases for $\alpha > \frac{1}{2}$ (extension thinning fluids). This is also supported by Fig. 1, where the extensional viscosity has been plotted against extension rate for two different De for different values of α. High extensional viscosities generate high spinline tension, which aid in stabilizing the spinning process. Low extensional viscosities generate low spinline tensions, which tend to destabilize the process. From the right figure in Fig. 2, we see that for extension thickening fluids ($\alpha = 0.2$), increase in De results in higher tension but lower tension sensitivity as seen in the dampening of the oscillations of the spinline tension. In the extension thinning case ($\alpha = 0.8$), increase in De results in lower tension but with increasing amplitude of the oscillations of the spinline tension showing unstable behaviour. For a detailed description of the physics behind the mechanism of draw resonance please refer to [13]. Moreover, from Eq. (3.8), it can be seen that keeping all other parameters constant, η is a decreasing function of α. Higher the value of α, lower is the extensional viscosity, generating lower spinline tensions which lead to more and more unstable behaviour. This is supported by Fig. 3, where cross-sectional areas and spinline tensions of the fibre at take-up for different values of α have been plotted against time.

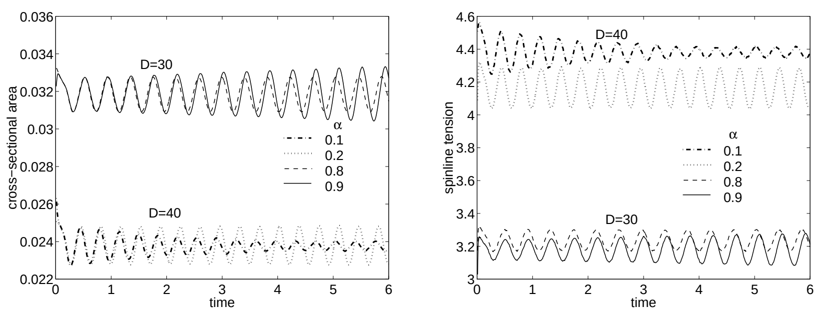

Figure 3. Transient response of the cross sectional area (left) and spinline tension (right) at take-up for different values of α.

Different values of draw ratio D have been taken for extension thickening and extension thinning fluids because for extension thickening fluids the critical draw ratio where the instability sets in is much higher than for extension thinning fluids. From the right figure we see that the spinline tensions decrease as α increases. Lower the spinline tensions, higher is the tension sensitivity leading to higher cross-sectional area sensitivity as seen in the left figure. Hence, Giesekus fluids with α close to 0 are the most stable and the stability decreases as α increases. By the above comparison of stability it is meant that the critical draw ratio where the draw resonance sets in would be the highest for Giesekus fluids with α approaching 0 and would keep decreasing as α increases. This is also consistent with the observation that extension thickening fluids show more stability than extension thinning fluids.

5 Conclusion

In this study, we have investigated the existence and uniqueness of stationary solutions of isothermal viscoelastic melt spinning equations. We have shown the relation between the boundary conditions of stress at the spinneret and the take-up velocity. The solution gives an insight into the rheological behaviour of the Giesekus fluid under elongation. For $\alpha = 0.5$, we have shown that the fluid exhibits Newtonian behaviour and serves as a borderline between extension thickening and extension thinning behaviour. To the knowledge of the authors, this is the first time that analytic stationary solutions to the isothermal viscoelastic melt spinning equations have been furnished. By simulating the complete system of non-stationary equations we have studied the effect of viscoelasticity on stability of the equilibrium solutions. We have also studied numerically the effect of the Giesekus model parameter on the stability of the spinning equations. Giesekus fluids with α close to 0 are the most stable and the stability decreases as α increases. The effect of increasing viscoelasticity on stability is such that for Giesekus fluids with $\alpha < 0.5$, increase in viscoelasticity leads to stability where as for $\alpha > 0.5$, increase in viscoelasticity leads to instability which is also in confirmation with previous studies done on different viscoelastic fluids.

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