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THE MAIN HASEMAN TYPE BOUNDARY VALUE PROBLEM FOR METAANALYTIC FUNCTION IN THE CASE OF CIRCULAR DOMAINS

B.F. FATULAEV

Smolensk State Pedagogical University

214000, Przevalskogo 4, Smolensk, Russia

E-mail: f_buba@mail.ru

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ABSTRACT

In the presented work a main Haseman type boundary value problem is investigated in the class of metaanalytic functions in the case of circular domains. With the help of Schwartz equation for circumference the initial problem can be reduced to the totality of two ordinary Haseman problems for analytic functions. Besides, the picture of solvability of the problem depending on values of indexes of coefficients of boundary value conditions is is investigated in this work.

1. THE STATEMENT OF THE PROBLEM

Let T^+ be a finite single-connected domain on the plane of the complex variable $z = x + iy$, bounded by the simple closed smooth curve L, prescribed by equation $t = x(s) + iy(s)$, where $x(s)$ and $y(s)$ are functions of arc s, satisfying condition H (Holder's condition) with their derivatives to the second order includingly (i.e. $L \in C^2_\mu$). As T^- we shall denote the complementation of $T^+ \cup L$ to full complex plane. For the determination we shall count, that the beginning of coordinates is in T^+, and as positive by-pass of the contour L we shall choose that by-pass, when domain T^+ stays left.

Let us remind [2], [3], [4], that function $F(z) = U(x, y) + iV(x, y)$ is called *metaanalytic* in domain T^+ (T^-), if in this domain it is a regular solution of the differential equation

$$\frac{\partial^2 F(z)}{\partial \bar{z}^2} + A_1(z)\frac{\partial F(z)}{\partial \bar{z}} + A_0(z)F(z) = 0, \tag{1}$$

where $\bar{z} = x - iy$, $\dfrac{\partial}{\partial \bar{z}} = \dfrac{1}{2}\left\{\dfrac{\partial}{\partial x} + i\dfrac{\partial}{\partial y}\right\}$ is a differential Cauchy-Riemann operator, and the coefficients $A_0(z), A_1(z)$ are piecewise analytic functions with the line of jump L, prescribed so:

$$A_k(z) = \begin{cases} a_k, & \text{if } z \in T^+, \\ a_k/z^{2-k}, & \text{if } z \in T^-, \quad k = 0, 1, \end{cases} \tag{1a}$$

a_0, a_1 are some complex constants here.

It is known (see, e.g., [4], p. 139-140), that if characteristical equation

$$\lambda^2 + a_1\lambda + a_0 = 0 \tag{2}$$

has one (repeated) root λ_0, then any metaanalytic function $F^+(z)$ in domain T^+ can be represented in the form

$$F^+(z) = [\varphi_0^+(z) + \bar{z}\varphi_1^+(z)]\exp\{\lambda_0\bar{z}\}, \tag{3}$$

where $\varphi_k^+(z)$ $(k = 0, 1)$ are analytic in T^+ functions.

By analogy, any metaanalytic function $F^-(z)$ in domain T^- is prescribed so:

$$F^-(z) = [\varphi_0^-(z) + \bar{z}\varphi_1^-(z)]\exp\{\lambda_0\bar{z}/z\}, \tag{4}$$

where $\varphi_k^-(z)$ $(k = 0, 1)$ are analytic in T^- functions.

Later on we will use terms and notations adopted in [4].

We shall call piecewise metaanalytic function with the line of jump L function $F(z)$, that in two domains, complementing each other to the full plane, T^+ and T^-, can be determined so:

$$F(z) = \begin{cases} F^+(z) = [\varphi_0^+(z) + \bar{z}\varphi_1^+(z)]\exp\{\lambda_0\bar{z}\}, & z \in T^+, \\ F^-(z) = [\varphi_0^-(z) + \bar{z}\varphi_1^-(z)]\exp\{\lambda_0\bar{z}/z\}, & z \in T^-, \end{cases} \tag{5}$$

where $\varphi_k^+(z)$ are analytic in T^+ and $\varphi_k^-(z)$ are analytic in T^- functions, $k = 0, 1$, λ_0 is a repeated root of the equation (2) and also there exist finite limits:

$$\lim_{z \to t \in L} F^+(z) = F^+(t), \qquad \lim_{z \to t \in L} F^-(z) = F^-(t).$$

We shall also call the metaanalytic function, prescribed by formula (5) vanishing in the infinity, if $\Pi\{\varphi_k^-; \infty\} \geq 1 + k$; $k = 0, 1$, where $\Pi\{\varphi_k^-, \infty\}$ is the order of the function $\varphi_k^-(z)$ at the point $z = \infty$.

Finally, we shall say, that piecewise metaanalytic function $F^{\pm}(z)$ belongs to the class $M_2(T^{\pm}) \cap H^{(p)}(L)$, if it extends unremittingly to the boundary L with its partial derivatives $\partial^{m+q} F^{\pm}(z)/\partial z^m \partial \bar{z}^q$ $(m+q \leq p)$ so, that the boundary values of this function and all the mentioned derivatives satisfy condition H by z.

Now let us consider the following boundary value problem:

Problem $\boldsymbol{H_{2,M}}$ (the main Haseman boundary value problem).
It is required to find all piecewise metaanalytic functions $F^{\pm}(z)$*, belonging to the class* $M_2(T^{\pm}) \cap H^{(2)}(L)$ *vanishing in the infinity and satisfying the boundary value conditions on* L

$$F^{+}[\alpha(t)] = G_0(t) \cdot F^{-}(t) + g_0(t), \tag{6}$$

$$\frac{\partial F^{+}[\alpha(t)]}{\partial n_{+}} = -G_1(t) \frac{\partial F^{-}(t)}{\partial n_{-}} + i \cdot t' \cdot g_1(t), \tag{7}$$

where $\partial/\partial n_{+}$ $(\partial/\partial n_{-})$ *is the derivative on interior (exterior) normal to* L*,* $L \in C^2_{\mu}$*,* $t' = dt/ds$*, and* $G_k(t), g_k(t)$ $(k = 0, 1)$ *are prescribed on* L *functions, and also* $G_k(t) \in H^{(3-k)}(L)$*,* $g_k(t) \in H^{(2-k)}(L)$ *and* $G_k(t) \neq 0$ *on* L*;* $\alpha(t)$ *is a shift function, preserving the orientation of contour* L *and besides* $\alpha'(t) \neq 0$*,* $\alpha(t) \in H^{(2)}(L)$.

Here in condition (7) multiplicants (-1) near $G_1(t)$ and it' near $g_1(t)$ are accordingly introduced for convenience in further notations.

Problem $\boldsymbol{H_{2,M}}$ in case of arbitrary smooth contour is investigated in detail in work [5], where it is proved, that the solution of problem $\boldsymbol{H_{2,M}}$ is reduced to sequential solving of *generalized* and *usual* Haseman type problems for analytic functions. In this article a significant particular case is considered, when $L = \{t : |t| = 1\}$. In this case, using Schwartz equation for circumference L, problem $\boldsymbol{H_{2,M}}$ can be reduced to the totality of two *usual* Haseman type problems for the classes of analytic functions.

2. THE SOLUTION OF PROBLEM $\boldsymbol{H_{2,M}}$

Let $L = \{t : |t| = 1\}$ and $T^{+} = \{z : |z| < 1\}$. We shall search for the solution of problem $\boldsymbol{H_{2,M}}$ in the form of (5). As $L = \{t : |t| = 1\}$ is a smooth contour, then we have (see, e.g., [1], p. 304)

$$\frac{\partial}{\partial n_{\pm}} = \pm i \left\{ t' \frac{\partial}{\partial t} - \bar{t}' \frac{\partial}{\partial \bar{t}} \right\}. \tag{8}$$

Besides on $L = \{t : |t| = 1\}$ the following equations are true:

$$\bar{t} = \frac{1}{t}, \qquad t' = it. \tag{9}$$

Taking into consideration (5), (8) and (9), we can rewrite accordingly boundary value conditions (6) and (7) so:

$$W^+[\alpha(t)] = \tilde{G}_0(t) \cdot W^-(t) + \tilde{g}_0(t), \tag{10}$$

$$V^+[\alpha(t)] = \tilde{G}_1(t) \cdot V^-(t) + \tilde{g}_1(t), \tag{11}$$

where

$$W^+(z) = z\varphi_0^+(z) + \varphi_1^+(z),\ W^-(z) = \varphi_0^-(z) + \frac{1}{z}\varphi_1^-(z), \tag{12}$$

$$\begin{gathered} V^+(z) = z^3\frac{d\varphi_0^+(z)}{dz} + z^2\frac{d\varphi_1^+(z)}{dz} + \lambda_0 z\varphi_0^+(z) + (\lambda_0 + z)\varphi_1^+(z), \\ V^-(z) = z\frac{d\varphi_0^-(z)}{dz} + \frac{d\varphi_1^-(z)}{dz} + \frac{\varphi_1^-(z)}{z}, \end{gathered} \tag{13}$$

$$\tilde{G}_0(t) = G_0(t) \cdot \alpha(t) \cdot \exp\left\{\lambda_0\left(\frac{1}{t^2} - \frac{1}{\alpha(t)}\right)\right\},$$

$$\tilde{G}_1(t) = G_1(t) \cdot \frac{[\alpha(t)]^3}{t \cdot \alpha'(t)} \cdot \exp\left\{\lambda_0\left(\frac{1}{t^2} - \frac{1}{\alpha(t)}\right)\right\},$$

$$\tilde{g}_0(t) = g_0(t) \cdot \alpha(t) \cdot \exp\left\{-\frac{\lambda_0}{\alpha(t)}\right\},\quad \tilde{g}_1(t) = g_1(t) \cdot \frac{[\alpha(t)]^3}{\alpha'(t)} \cdot \exp\left\{-\frac{\lambda_0}{\alpha(t)}\right\}.$$

It is easy to notice from the last formulae, that

$$\tilde{\varkappa}_k = Ind\,\tilde{G}_k(t) = \varkappa_k + k + 1, \tag{14}$$

where $\varkappa_k = Ind\,G_k(t)$ $(k = 0, 1)$.

Equation (10) is the boundary value condition of usual Haseman type problem relatively to the vanishing in the infinity piecewise analytic function $W(z) = \{W^+(z), W^-(z)\}$. The solution of problem (10), as it is known (see, e.g., [4], §21) if $\tilde{\varkappa}_0 = Ind\,\tilde{G}_0(t) \geq 0$ is prescribed by the formulae

$$W^+(z) = X_0^+(z)\left\{\frac{1}{2\pi i}\int\limits_L \frac{\tilde{g}_0[\beta(\tau)]}{X_0^+(\tau)[\tau - z]}d\tau + \int\limits_L R_0^+(z,\tau)\tilde{g}_0[\beta(\tau)]d\tau\right.$$

$$\left. + \sum_{k=0}^{\tilde{\varkappa}_0 - 1} c_k d_{1k}^+(z)\right\},\quad z \in T^+, \tag{15a}$$

$$W^-(z) = X_0^-(z)\left\{\frac{1}{2\pi i}\int\limits_L \frac{\tilde{g}_0(\tau)}{X_0^+[\alpha(\tau)]} \cdot \frac{d\tau}{\tau - z} + \int\limits_L R_0^-(z,\tau)\tilde{g}_0(\tau)d\tau\right.$$

$$+\sum_{k=0}^{\tilde{\varkappa}_0-1} c_k d_{1k}^-(z)\Bigg\}, \quad z \in T^-, \tag{15b}$$

where $X_0^{\pm}(z)$ are canonical functions of problem (10), $\beta(t)$ is the function, which is reversal to $\alpha(t)$,

$$R_0^+(z,\tau) = \frac{[X_0^+(\tau)]^{-1}}{2\pi i}\int\limits_L \frac{\beta'(\tau)R[\beta(\tau_1),\tau]}{\tau_1 - z}d\tau_1, \quad z \in T^+,$$

$$R_0^-(z,\tau) = \frac{[X_0^+(\alpha(\tau))]^{-1}}{2\pi i}\int\limits_L \frac{R(\tau_1,\tau)}{\tau_1 - z}d\tau_1, \quad z \in T^-,$$

$$d_{1k}^+(z) = \frac{1}{2\pi i}\int\limits_L \frac{[\beta(\tau)]^k}{\tau - z}d\tau + \frac{1}{2\pi i}\int\limits_L \left[\int\limits_L R(\beta(\tau),\tau_1)\tau_1^k d\tau_1\right]\frac{d\tau}{\tau - z}, \quad z \in T^+,$$

$$d_{1k}^-(z) = z^k + \frac{1}{2\pi i}\int\limits_L \left[\int\limits_L R(\tau,\tau_1)\tau_1^k d\tau_1\right]\frac{d\tau}{\tau - z}, \quad z \in T^-, \tag{15c}$$

$R(t,\tau)$ is a resolvent of the kern of the integral equation

$$\psi(t) + \frac{1}{2\pi i}\int\limits_L \left[\frac{\alpha'(\tau)}{\alpha(\tau)-\alpha(t)} - \frac{1}{\tau - t}\right]\psi(\tau)d\tau = \frac{\tilde{g}_0(t)}{X_0^+[\alpha(t)]}, \tag{15d}$$

and c_k $(k = 0,1,\dots,\tilde{\varkappa}_0 - 1)$ are arbitrary complex constants.

But if $\tilde{\varkappa}_0 < 0$, then the solution of problem (10) will also be expressed by formulae (15a)-(15c), where we must insert $c_k = 0$ $(k = 0,1,\dots,\tilde{\varkappa}_0 - 1)$ with the observance of the following $|\tilde{\varkappa}_0|$ conditions of the existence of solution (see [4], §21):

$$\int\limits_L h_{0k}(\tau)\tilde{g}_0(\tau)d\tau = 0, \qquad k = 1,2,\dots,\tilde{\varkappa}_0, \tag{16}$$

where

$$h_{0k}(\tau) = \frac{1}{X_0^+[\alpha(\tau)]}\left[\tau^{k-1} + \int\limits_L R(\tau,\tau_1)\tau_1^{k-1}d\tau_1\right]. \tag{16a}$$

Further on, solving by analogy usual Haseman type problem (11) for vanishing in infinity piecewise analytic functions, we find analytic functions $V^+(z)$ and $V^-(z)$.

Using the solutions of problems (10) and (11) one can find the analytic components of the required metaanalytic function.

Really, by equalities (12) and (13), the following two systems of equalities are to be true:

$$\begin{cases} z\varphi_0^+(z)+\varphi_1^+(z)=W^+(z), \\ z^3\dfrac{d\varphi_0^+(z)}{dz}+z^2\dfrac{d\varphi_1^+(z)}{dz}+\lambda_0 z\varphi_0^+(z)+(\lambda_0+z)\varphi_1^+(z)=V^+(z), \end{cases} \tag{17}$$

$$\begin{cases} \varphi_0^-(z)+\dfrac{1}{z}\varphi_1^-(z)=W^-(z), \\ z\dfrac{d\varphi_0^-(z)}{dz}+\dfrac{d\varphi_1^-(z)}{dz}+\dfrac{1}{z}\varphi_1^-(z)=V^-(z). \end{cases} \tag{18}$$

Solving systems (17) and (18) relatively to $\varphi_k^+(z)$ and $\varphi_k^-(z)$ $(k=0,1)$, we will obtain:

$$\begin{aligned} \varphi_0^+(z)&=\frac{1}{2}\frac{dW^+(z)}{dz}+\frac{\lambda_0+z}{2z^2}W^+(z)-\frac{V^+(z)}{2z^2}, \\ \varphi_1^+(z)&=\frac{z-\lambda_0}{2z}W^+(z)+\frac{V^+(z)}{2z}-\frac{z}{2}\frac{dW^+(z)}{dz}, \end{aligned} \tag{19}$$

$$\begin{aligned} \varphi_0^-(z)&=W^-(z)-\frac{1}{2}V^-(z)+\frac{z}{2}\frac{dW^-(z)}{dz}, \\ \varphi_1^-(z)&=\frac{z}{2}V^-(z)-\frac{z^2}{2}\frac{dW^-(z)}{dz}. \end{aligned} \tag{20}$$

As functions $W^+(z)$, $V^+(z)$ are analytic in circle $T^+=\{z:\,|z|<1\}$, then we can expand them near the point $z=0$ in series, i.e. we can rewrite them in the form (wee [6], p. 130):

$$W^+(z)=\sum_{k=0}^{\infty}a_k\cdot z^k,\quad V^+(z)=\sum_{k=0}^{\infty}b_k\cdot z^k,\quad z\in T^+, \tag{21}$$

where

$$a_k=\frac{1}{2\pi i}\int_L W^+(\tau)\tau^{-k-1}d\tau,\quad b_k=\frac{1}{2\pi i}\int_L V^+(\tau)\tau^{-k-1}d\tau, \tag{21a}$$

$W^+(\tau), V^+(\tau)$ are boundary values of functions $W^+(z)$ and $V^+(z)$ accordingly.

And also functions $W^-(z)$, $V^-(z)$ are analytic out of singular circle T^+ and vanish in infinity, it means that their expansions in series near infinitely distant point will have the form (see, e.g., [6], p. 153):

$$W^-(z)=\sum_{k=1}^{\infty}\tilde{a}_k\cdot z^{-k},\quad V^-(z)=\sum_{k=1}^{\infty}\tilde{b}_k\cdot z^{-k},\quad z\in T^-, \tag{22}$$

where

$$\tilde{a}_k = \frac{1}{2\pi i}\int\limits_L W^-(\tau)\tau^{k-1}d\tau, \quad \tilde{b}_1 = \frac{1}{2\pi i}\int\limits_L V^-(\tau)\tau^{k-1}d\tau, \tag{22a}$$

$W^-(\tau), V^-(\tau)$ are boundary values of functions $W^-(z)$ and $V^-(z)$ accordingly.

By the condition of the problem functions $\varphi_k^+(z),\ k = 0, 1$, must be analytic in circle T^+, and functions $\varphi_k^-(z)$, $k = 0, 1$, analytic in domain T^- and vanishing in infinity, moreover function $\varphi_1^-(z)$ is to have a zero of the order not less than 2 in infinity. Then, substituting the expansions of functions $W^\pm(z)$, $V^\pm(z)$ and their derivatives, found by formulae (21), (22) into the equalities (19), (20) we will obtain conditions, which in addition must be satisfied by the solutions of problems (10) and (11):

$$\left\{\begin{array}{c} \lambda_0 a_0 - b_0 = 0, \\ \lambda_0 a_1 + a_0 - b_1 = 0, \\ \tilde{a}_1 + \tilde{b}_1 = 0, \\ 2\tilde{a}_2 + \tilde{b}_2 = 0. \end{array}\right. \tag{23}$$

If conditions (23) are satisfied, we can prescribe the solution of problem $\boldsymbol{H_{2,M}}$ by the formula

$$F(z) = \left\{\begin{array}{ll} F^+(z) = [\varphi_0^+(z) + \bar{z}\varphi_1^+(z)]\exp\{\lambda_0\bar{z}\}, & z \in T^+, \\ F^-(z) = [\varphi_0^-(z) + \bar{z}\varphi_1^-(z)]\exp\{\lambda_0\bar{z}/z\}, & z \in T^-, \end{array}\right. \tag{24}$$

where $\varphi_0^\pm(z)$, $\varphi_1^\pm(z)$ can be defined by equalities (19), (20).

Now let us investigate the picture of solvability of problem $\boldsymbol{H_{2,M}}$.

From the results of the previous investigation we can conclude that the picture of solvability of problem $\boldsymbol{H_{2,M}}$ consists of the pictures of solvability of problems (10) and (11), which in their turn depend on the values of indexes $\tilde{\varkappa}_0 = Ind\,\tilde{G}_0(t)$ and $\tilde{\varkappa}_1 = Ind\,\tilde{G}_1(t)$. Hence 4 cases are possible here:

a) $\tilde{\varkappa}_0 \geq 0$, $\tilde{\varkappa}_1 \geq 0$. In this case problems (10) and (11) are solvable without conditions and the common solutions contain $\tilde{\varkappa}_0$ and $\tilde{\varkappa}_1$ of the arbitrary complex constants accordingly. Consequently conditions (23) are the system of algebraic equations relative to $\tilde{\varkappa}_0 + \tilde{\varkappa}_1$ unknowns $c_0, \ldots, c_{\tilde{\varkappa}_0 - 1}, \delta_0, \ldots, \delta_{\tilde{\varkappa}_1 - 1}$. Let this system be solvable and r is the rank of its main matrix. Then from constants c_k, δ_j $(k = 0, \ldots, \tilde{\varkappa}_0 - 1,\ j = 0, \ldots, \tilde{\varkappa}_1 - 1)$ exactly r are expressed by the other $\tilde{\varkappa}_0 + \tilde{\varkappa}_1 - r$. Then the common solution of problem $\boldsymbol{H_{2,M}}$ depends on $\tilde{\varkappa}_0 + \tilde{\varkappa}_1 - r$ arbitrary complex constants, where $0 \leq r \leq \min\{4, \tilde{\varkappa}_0 + \tilde{\varkappa}_1\}$.

b) $\tilde{\varkappa}_0 < 0$, $\tilde{\varkappa}_1 \geq 0$. In this case problem (11) is solvable without conditions and its common solution depends on $\tilde{\varkappa}_1$ arbitrary complex constants δ_k $(k = 0, 1 \ldots, \tilde{\varkappa}_1 - 1)$. And also problem (10) has singular solution if $|\tilde{\varkappa}_0|$ conditions of solvability of type (16) are satisfied. If conditions (16) are satisfied, then conditions (23) are the system of algebraic equations relative to unknowns

$\delta_0, \dots, \delta_{\tilde{\varkappa}_1 - 1}$. If this system is solvable the common solution of problem $\boldsymbol{H_{2,M}}$ depends on $\tilde{\varkappa}_1 - r$ arbitrary complex constants, where r is the rank of a definite matrix, and also $0 \le r \le \min\{4, \tilde{\varkappa}_1\}$ here.

c) $\tilde{\varkappa}_0 \ge 0$, $\tilde{\varkappa}_1 < 0$. In this case problem (10) is solvable without conditions and its common solution depends on $\tilde{\varkappa}_0$ arbitrary constants $c_0, \dots, c_{\tilde{\varkappa}_0 - 1}$. At the same time problem (11) is solvable if the following $|\tilde{\varkappa}_1|$ conditions are satisfied:

$$\int\limits_L h_{1k}(\tau) \tilde{g}_1(\tau) d\tau = 0, \qquad k = 1, 2, \dots, -\tilde{\varkappa}_1, \tag{25}$$

where

$$h_{1k}(\tau) = \frac{1}{X_1^+[\alpha(\tau)]} \left[\tau^{k-1} + \int\limits_L R(\tau, \tau_1) \tau_1^{k-1} d\tau_1 \right],$$

$X_1^+(z)$ is the canonical function of the problem, and $R(t, \tau)$ is the same as in (15a)-(15d). Besides it will have singular solution. Hence, if conditions (25) are satisfied, then conditions (23) are the system of algebraic equations relative to constants $c_0, \dots, c_{\tilde{\varkappa}_0 - 1}$. If the system is solvable, the common solution of problem $\boldsymbol{H_{2,M}}$ depends on $\tilde{\varkappa}_0 - r$ arbitrary complex constants, where r is the rank of a definite matrix, and also $0 \le r \le \min\{4, \tilde{\varkappa}_0\}$.

d) $\tilde{\varkappa}_0 < 0$, $\tilde{\varkappa}_1 < 0$. In this case problems (10) and (11) have singular solutions if $|\tilde{\varkappa}_0|$ conditions of solvability (16) and $|\tilde{\varkappa}_1|$ condition of solvability (25) are satisfied accordingly. Then, if all these conditions and conditions (23) are satisfied at the same time, then problem $\boldsymbol{H_{2,M}}$ will also have singular solution, prescribed by formula (24).

So, in case of circular domain, we have obtained the following result.

Theorem 2.1. *Let characteristical equation* (2) *have one (repeated) root* λ_0 *and contour* $L = \{t : |t| = 1\}$. *Then:*

1) if $\tilde{\varkappa}_0 \ge 0$ *and* $\tilde{\varkappa}_1 \ge 0$*, then for the solvability of problem* $\boldsymbol{H_{2,M}}$ *satisfying of conditions* (23) *is necessary and sufficient, and, if these conditions are fulfilled the common solution of problem* $\boldsymbol{H_{2,M}}$ *is prescribed by formula* (24), *and it linearly depends on* $\tilde{\varkappa}_0 + \tilde{\varkappa}_1 - r$ *arbitrary complex constants, where* r *is the rank of a definite matrix* $(0 \le r \le \min\{4, \tilde{\varkappa}_0 + \tilde{\varkappa}_1\})$*;*

2) if $\tilde{\varkappa}_0 < 0$ *and* $\tilde{\varkappa}_1 \ge 0$*, then for the solvability of problem* $\boldsymbol{H_{2,M}}$ *simultaneous satisfying of conditions* (16) *and* (23) *is necessary and sufficient, and, if these conditions are fulfilled the common solution of problem* $\boldsymbol{H_{2,M}}$ *is prescribed by formula* (24), *and it linearly depends on* $\tilde{\varkappa}_1 - r$ *arbitrary complex constants, where* r *is the rank of a definite matrix* $(0 \le r \le \min\{4, \tilde{\varkappa}_1\})$*;*

3) if $\tilde{\varkappa}_0 \ge 0$ *and* $\tilde{\varkappa}_1 < 0$*, then for the solvability of problem* $\boldsymbol{H_{2,M}}$ *simultaneous satisfying of conditions* (23) *and* (25) *is necessary and sufficient, and, if these conditions are fulfilled the common solution of problem* $\boldsymbol{H_{2,M}}$ *is prescribed by formula* (24), *and it linearly depends on* $\tilde{\varkappa}_0 - r$ *arbitrary complex constants, where* r *is the rank of a definite matrix* $(0 \le r \le \min\{4, \tilde{\varkappa}_0\})$*;*

4) at last, if $\tilde{\varkappa}_0 < 0$ *and* $\tilde{\varkappa}_1 < 0$*, then for the solvability of problem* $\boldsymbol{H_{2,M}}$ *simultaneous satisfying of conditions* (16), (23) *and* (25) *is necessary and sufficient and if these conditions are satisfied, then it will have the singular solution, prescribed by formula* (24).

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METAANALIZINIŲ FUNKCIJŲ PAGRINDINIS HASEMANO TIPO KRAŠTINIS UŽDAVINYS SKRITULINIŲ SRIČIŲ ATVEJU

V.F. Fatulaev

Darbe nagrinėjimas pagrindinis Hasemano tipo matanalizinių funkcijų kraštinis uždavinys $\boldsymbol{H_{2,M}}$ skritulinėje srityje. Bet kurios srities (su glodžiu kontūru) atveju $\boldsymbol{H_{2,M}}$ uždavinys išnagrinėtas kitame autoriaus darbe, kuriame parodyta, kad šį uždavinį galima suvesti į du atskirus analizinių funkcijų uždavinius, būtent, į taip vadinamus apibendrintąjį ir paprastąjį Hasemano uždavinius. Šiame darbe įrodyta, kad skritulinės srities atveju pagrindinį $\boldsymbol{H_{2,M}}$ uždavinį galima suvesti į du atskirus įprastuosius Hasemano uždavinius analizinių funkcijų klasėje. Be to, yra parodyta, kad nagrinėjamojo uždavinio išsprendžiamumas priklauso nuo kraštinių sąlygų indekso.