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ON THE FIRST BOUNDARY VALUE PROBLEM FOR THE CLASS OF ELLIPTIC SYSTEMS DEGENERATING AT AN INNER POINT

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ABSTRACT

The Dirichlet type problem for the weakly related elliptic systems of the second order degenerating at an inner point is discussed. Existence and uniqueness of the solution in the Holder class of the vector-functions is proved.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

Let $\mathcal{D} \subset \mathbb{R}^n$ be bounded domain with a boundary $S \in C^{2,\alpha}(0 < \alpha \leq 1)$ and let the origin $x = 0$ be an inner point of domain $\mathcal{D}$. We consider in $\mathcal{D}$ the system of equations

$$Lu := \sum_{i,j=1}^{n} A_{ij}(x) u_{x_i x_j} + \sum_{i=1}^{n} B_i(x) u_{x_i} - C(x)u = f(x), \qquad (1.1)$$

where $A_{ij} = \mathrm{diag}(a_{ij}^{(1)}, \ldots, a_{ij}^{(m)}), B_i = \mathrm{diag}(b_i^{(1)}, \ldots, b_i^{(m)})$ and $C = \big(c_{kl}\big)$ are real continuous square matrices of order m, $f = \big(f_1, \ldots, f_m\big)$ and $u = \big(u_1, \ldots, u_m\big)$ are given and unknown vector-columns, respectively.

Henceforth, we shall use the following notations: $R = \max_{x \in \overline{\mathcal{D}}} |x|$, $\mathcal{D}_0 = \mathcal{D} \backslash \{x =$

$0\}, \sum_\delta^0 = \{x : 0 < |x| < \delta\}$, $\mathcal{D}_\delta = \mathcal{D} \setminus \overline{\sum_\delta^0}$, $S_\delta = \{x : |x| = \delta\} (\delta < \mathrm{dist}(0, S))$. We shall denote by r the length of vector $x = (x_1, \ldots, x_n)$ and by $|\circ|_{l,\alpha;\mathcal{D}}$ and $|\circ|_{l;\mathcal{D}}$ norms in the corresponding spaces $C^{l,\alpha}(\overline{\mathcal{D}})$ and $C^l(\overline{\mathcal{D}})$, where $l \geq 0$ is an integer.

We assume that the following conditions are fulfilled:

a) there exist continuous functions μ_1 and μ_2, and a number $\gamma_1 > 0$ such that $0 < \mu_1(r) \leq \mu_2(r)$ for $r \in (0, R], \mu_2(r) = O(r^{2+\gamma_1})$ as $r \to 0$, and

$$\mu_1(r)|\xi|^2 \leq \sum_{i,j=1}^{n} a_{ij}^{(k)}(x)\xi_i\xi_j \leq \mu_2(r)|\xi|^2, \quad k = \overline{1,m}, \tag{1.2}$$

for each $x \in \overline{\mathcal{D}}$ and for each $\xi = (\xi_1, \ldots, \xi_n) \in \mathbb{R}^n$;

b) the relations

$$b_i^{(k)}(x) = O(r^{1+\gamma_2}), \quad i = \overline{1,n},\ k = \overline{1,m}, \tag{1.3}$$

as $r \to 0$ with any $\gamma_2 > 0$ hold;

c) there exist a number $\nu > 0$ such that

$$c_{kk}(x) - \sum_{l \neq k} |c_{kl}(x)| \geq \nu, \quad k = \overline{1,m}, \tag{1.4}$$

for each $x \in \mathcal{D}$.

Observe, that in accordance with (1.2) system (1.1) is elliptic in $\mathcal{D}_0$ in the sense of Petrovskii and is strongly (nonregularly) degenerate at the origin $x = 0$.

We introduce the class of vector-functions

$$C_1^{2,\alpha}(\mathcal{D}) = \{u : u \in C^{2,\alpha}(\overline{\mathcal{D}}_\delta) \quad \forall\delta > 0,\ |u| < \infty \quad in\ \mathcal{D}_0\}.$$

Due to degeneracy of system (1.1) it has the solutions both bounded and unbounded at the point $x = 0$ (see, e.g. [2], [3], [7]). Therefore, one can discuss the Dirichlet type problem for (1.1) in the class of functions bounded in $\mathcal{D}_0$. Such a problem for scalar equation (1.1) with $A_{ij}(x) = \delta_{ij}|x|^{2+\gamma_1}$ (δ_{ij} is the Kroenecker symbol) and $B_i(x) \equiv 0$ $(i, j = \overline{1,n})$ is solved in [7]. This problem in the case the main part of system (1.1) is like in [7] is considered in [4]. The case of system (1.1) where coefficients $B_i(x)$ are not vanishing at the point $x = 0$ is investigated in [5]. Sufficient conditions are indicated there under which Dirichlet type problem has the unique solution in the class $C_1^{2,\alpha}(\mathcal{D})$ defined above. In this article we shall solve this problem for system (1.1) in the case of coefficients $B_i(x)$ are vanishing fastly enough as $r \to 0$ (see (1.3)).

Thus, we consider the following problem:

$$Lu = f \text{ in } \mathcal{D}_0, \tag{1.5}$$
$$u = g \text{ on } S, \tag{1.6}$$
$$u \in C_1^{2,\alpha}(\mathcal{D}), \tag{1.7}$$

where $g = (g_1, \dots, g_n)$ is given continuous vector-function.

As the main result we shall prove the following statement.

Theorem 1.1. *Let the imbeddings A_{ij} and $B_i (i,j = \overline{1,n}), C$ and $f \in C^{0,\alpha}(\overline{\mathcal{D}})$, and $g \in C^{2,\alpha}(S)$, hold. If conditions (1.2) – (1.4) are fulfilled, then there exists the unique solution of problem (1.5) – (1.7).*

2. AUXILIARIES

Lemma 2.1. *Let $A_{ij}, B_i (i,j = \overline{1,n}), C$ and f be continuous in closed domain $\overline{\mathcal{D}}$ and let $u = (u_1, \dots, u_m)$ be a solution of (1.1) from $C^2(\mathcal{D}_\delta) \cap C^0(\overline{\mathcal{D}}_\delta)$. If condition (1.4) is fulfilled, the estimate*

$$|u_j|_{0;\mathcal{D}_\delta} \le \max_{1 \le i \le m} \{\max_{\partial \mathcal{D}_\delta} |u_i|,\ \nu^{-1} |f_i|_{0;\mathcal{D}}\} \tag{2.1}$$

holds for every $j = \overline{1,m}$.

Proof. It is easily seen that every component u_j of vector-function u satisfies either inequality

$$|u_j|_{0;\mathcal{D}_\delta} > \max_{\partial \mathcal{D}_\delta} |u_j| \tag{2.2}$$

or equality

$$|u_j|_{0;\mathcal{D}_\delta} = \max_{\partial \mathcal{D}_\delta} |u_j|. \tag{2.3}$$

If (2.3) for every $j = \overline{1,m}$ holds, then estimate (2.1) is true, obviously. Let some components u_j do not satisfy (2.3). Assume without a loss of generality that these are $u_1, \dots, u_{m_0}$, where $m_0 \le m$. In such a case each of functions $|u_j|, j = \overline{1,m_0}$, attain its positive absolute maximum at an inner point $x^j \in \mathcal{D}_\delta$, correspondingly. Choose the largest one from the number set $\{|u_j(x^j)|\}, j = \overline{1,m_0}$. Let this number be $u^k := \big|u_k(x^k)\big|$. If

$$u^k \le \max_{\partial \mathcal{D}_\delta} |u_i|, \quad i = \overline{m_0 + 1, m}, \tag{2.4}$$

then we get that

$$|u_j|_{0;\mathcal{D}_\delta} \le \max_{1 \le i \le m} \{\max_{\partial \mathcal{D}_\delta} |u_i|\}, \quad j = \overline{1,m},$$

i.e. in this case estimate (2.1) holds.

Let us assume that u^k does not satisfy (2.4). In this case the proof of lemma will be complete, indeed, if we shall show that the condition

$$u^k > \max_{\partial\mathcal{D}_\delta} |u_i|, \quad i = \overline{m_0+1, m}, \tag{2.5}$$

implies the estimate

$$u^k \le \nu^{-1} \max_{1\le i\le m} \{|f_i|_{0;\mathcal{D}}\}. \tag{2.6}$$

Let $u_k(x^k)$ be greater than zero. Observe, that then x^k is the maximum point of function u_k. Therefore,

$$(u^k)_{x_i}\big|_{x=x^k} = 0, \quad i = \overline{1,n}, \tag{2.7}$$

and according to ellipticity of system (1.1) the inequality

$$\sum_{i,j=1} a_{ij}^{(k)}(x)(u_k)_{x_ix_j}\big|_{x=x^k} \le 0$$

holds. Due to it and (2.7) we obtain from (1.1) that

$$u_k(x^k)c_{kk}(x^k) + \sum_{l\ne k} c_{kl}(x^k)u_l(x^k) \le -f_k(x^k). \tag{2.8}$$

Since $u^k \ge |u_l(x^k)|$ for $l = \overline{1, m_0}$ (in accordance with the choice of u^k) and (2.5) holds, we get in view of (1.4) the estimate

$$u_k(x^k)c_{kk}(x^k) + \sum_{l\ne k} c_{kl}(x^k)u_l(x^k) \ge \nu u^k.$$

Hence, taking into account (2.8) we obtain that

$$u^k \le -f_k(x^k)/\nu \le \nu^{-1} \max_{1\le i\le m} \{|f_i|_{0;\mathcal{D}}\},$$

i.e. estimate (2.6) holds.

Letting $u_k(x^k)$ be less than zero one can show in quite a similar way that under assumption (2.5) and in view of (1.4) estimate (2.6) holds, too. Lemma is proved. ■

Remark 2.1. If $m = 1$ and $f \equiv 0$, estimate (2.1) represents the well known maximum principle for scalar elliptic equations.

Now we shall prove one property of the operator

$$L_k := \sum_{i,j=1}^{n} a_{ij}^{(k)}(x)\partial^2/\partial x_i \partial x_j + \sum_{i=1}^{n} b_i^{(k)}(x)\partial/\partial x_i - c_{kk}(x),$$

where k is arbitrarily fixed, $1 \le k \le m$.

Lemma 2.2. *Let $a_{ij}^{(k)}, b_i^{(k)} (i,j = \overline{1,n})$ and $c_{kk} \in C^0(\overline{\mathcal{D}})$. Let conditions (1.2), (1.3) be fulfilled and let a number β be such that $\beta \in (0, \gamma')$ with $\gamma' = \min\{\gamma_1, \gamma_2\}$. Then there exists a positive constant N_k such that*

$$L_k r^{-\beta} \le -c_{kk}(x) r^{-\beta} + N_k r^{\gamma' - \beta} \tag{2.9}$$

for each $x \in \mathcal{D}_0$.

Proof. Observe that condition (1.2) yields the relations

$$\sum_{i=1}^{n} a_{ii}^{(k)}(x) = O(r^{2+\gamma_1}), \quad \sum_{i,j=1}^{n} a_{ij}^{(k)}(x) x_i x_j = O(r^{4+\gamma_1}) \tag{2.10}$$

as $r \to 0$.

By the direct calculation we obtain that

$$L_k r^{-\beta} = -r^{-\beta}\Big(c_{kk}(x) + \psi_k(x)\Big), \tag{2.11}$$

where

$$\psi_k(x) = \beta(\beta - 2) r^{-4} \sum_{i,j=1}^{n} a_{ij}^{(k)} x_i x_j + \beta r^{-2} \sum_{i=1}^{n} \Big(a_{ii}^{(k)}(x) + x_i b_i^{(k)}(x)\Big).$$

It is easily seen that in view of (1.3) and (2.10) and due to the smoothness of $a_{ij}^{(k)}$ and $b_i^{(k)}$ there exists

$$N_k := \sup_{\mathcal{D}_0} \big(|\psi_k(x)|/r^{\gamma'}\big),$$

obviously. This jointly with (2.11) imply (2.9). Lemma is proved. ■

Theorem 2.1. *Let imbeddings $A_{ij}, B_i (i,j) = \overline{1,n}$ and $C \in C^0(\overline{\mathcal{D}})$ hold and conditions (1.2) – (1.4) be fulfilled. If $u = (u_1, \dots, u_m) \in C^2(\mathcal{D}_0) \cap C^0(\mathcal{D}_0 \cup S)$ is a solution of system $Lu = 0$ bounded in $\mathcal{D}_0$ and satisfying condition $u\big|_S = 0$, then $u = 0$ everywhere in domain $\mathcal{D}_0$.*

Proof. Introduce the function

$$w(r) = M + r^{-\beta}$$

with the constant M such that

$$M \geq N_k R^{\gamma' - \beta}/\nu \text{ for } \quad k = \overline{1, m}, \tag{2.12}$$

where β, γ' and N_k are from lemma 2.2.

Let $\varepsilon > 0$ be arbitrarily fixed number. Since $w(r) > 0$ for $r > 0$ and $w(r) \to +\infty$ as $r \to 0$, we obtain that

$$|u_k(x)| \leq \varepsilon w(r) \text{ for } k = \overline{1, m} \tag{2.13}$$

in $\sum_{\delta^*}^0 \cup S_{\delta^*}$ with $\delta^* = \big(\varepsilon/|\kappa - \varepsilon M|\big)^{1/\beta}$, where $\kappa = \sup\limits_{\mathcal{D}_0} |u|$.

We shall show that inequality (2.13) holds everywhere in $\mathcal{D}_{\delta^*}$, too. This will imply that (2.13) holds everywhere in $\mathcal{D}_0$. Thus, due to arbitrariness of ε we will get $u = 0$ in $\mathcal{D}_0$, i.e. theorem will be proved.

With that and in view let us introduce the vector-function $v = (v_1, \ldots, v_m)$ by formula

$$u(x) = w(r) v(x). \tag{2.14}$$

Putting (2.14) into system $Lu = 0$ we obtain that v is the solution from $C^2(\mathcal{D}_0) \cap C^0(\mathcal{D}_0 \cup S)$ of the system

$$\tilde{L} v := \sum_{i,j=1}^{n} \tilde{A}_{ij}(x) v_{x_i x_j} + \sum_{i=1}^{n} \tilde{B}_i(x) v_{x_i} - \tilde{C}(x) v = 0 \tag{2.15}$$

with the matrix-coefficients

$$\begin{aligned} \tilde{A}_{ij} &= \operatorname{diag}(\tilde{a}_{ij}^{(1)}, \ldots, \tilde{a}_{ij}^{(m)}), \\ \tilde{B}_i &= \operatorname{diag}(\tilde{b}_i^{(1)}, \ldots, \tilde{b}_i^{(m)}), \\ \tilde{C} &= (\tilde{c}_{kl}), \quad k \text{ and } l = \overline{1, m}, \end{aligned}$$

where

$$\begin{aligned} &\tilde{a}_{ij}^{(k)}(x) = \big(1 + M r^{\beta}\big) a_{ij}^{(k)}(x), \\ &\tilde{b}_i^{(k)}(x) = \big(1 + M r^{\beta}\big) b_i^{(k)}(x), \\ &\tilde{c}_{kl}(x) = \begin{cases} (1 + M r^{\beta}) c_{kl}(x), & \text{if } k \neq l, \\ M r^{\beta} c_{kk}(x) - r^{\beta} L_k r^{-\beta}, & \text{if } k = l. \end{cases} \end{aligned}$$

Obviously, the elements $\tilde{a}_{ij}^{(k)}$ of matrices $\tilde{A}_{ij}^{(k)}(i,j=\overline{1,n})$ satisfy condition (1.2) with functions $\tilde{\mu}_i(r)=(1+Mr^{\beta})\mu_i(r)$ instead of functions $\mu_i(r), i=1,2$. Moreover, since

$$\sum_{i,j=1}^{n} a_{ij}^{(k)} x_j = O(r^{3+\gamma_1}) \text{ as } r\to 0,$$

we get due to (1.3) the relation

$$\tilde{b}_i^{(k)}(x)=O(r^{1+\gamma'}) \quad \text{as } r\to 0.$$

Hence, the elements $\tilde{b}_i^{(k)}, k=\overline{1,m}$, of every matrix $\tilde{B}_i, i=\overline{1,n}$, satisfy the condition similar to (1.3).

Observe, that the limit values of $\tilde{a}_{ij}^{(k)}(x)$ and $\tilde{c}_{kl}(x)$ as $r\to 0$ coincide with the corresponding limits of functions $\tilde{a}_{ij}^{(k)}(x)$ and $\tilde{c}_{kl}(x)$. Besides that, $\tilde{b}_i^{(k)}(x)\sim b_i^{(k)}(x)$ as $r\to 0$, evidently. Therefore, matrices $\tilde{A}_{ij}, \tilde{B}_i(i,j=\overline{1,n})$ and $\tilde{C}$ are continuous at the point $x=0$.

Now we shall show that

$$\tilde{c}_{kk}(x)-\sum_{l\neq k}|\tilde{c}_{kl}(x)|\geq \nu, \quad k=\overline{1,m}, \tag{2.16}$$

for each $x\in\mathcal{D}$, i.e. matrix $\tilde{C}$ satisfies condition (1.4).

According to (2.9) we get the inequality

$$\tilde{c}_{kk}(x)\geq(1+Mr^{\beta})c_{kk}(x)-N_k r^{\gamma'}, \quad x\in\mathcal{D},$$

which jointly with (1.4) implies the estimate

$$\tilde{c}_{kk}(x)-\sum_{l\neq k}|\tilde{c}_{kl}(x)|\geq \nu(1+Mr^{\beta})-N_k r^{\gamma'},\ x\in\mathcal{D}. \tag{2.17}$$

Due to (2.12) we have

$$\nu M r^{\beta}-N_k r^{\gamma'}\geq N_k r^{\gamma'}\Big(\big(r/R\big)^{\beta-\gamma'}-1\Big)\geq 0$$

for each $r\in[0,R]$, i.e. (2.17) yields (2.16).

Thus, the coefficients of system (2.15) satisfy the conditions of lemma 2.1. According to it and in view of property $v\big|_S=(u/w)\big|_S=0$ we get from (2.1) the estimate

$$|v_j|_{0;\mathcal{D}_\delta^*}\leq \max_{1\leq i\leq m}\{\max_{S_\delta^*}|v_i|\},\ j=\overline{1,m},$$

i.e. the estimate $|v_j|_{0;\mathcal{D}_\delta^*}\leq\varepsilon$, because $|v_i=|u_i|/w\leq\varepsilon$ on S_{δ^*}. Therefore, $|u_k(x)|\leq\varepsilon w(r), k=\overline{1,m}$, in $\mathcal{D}_{\delta^*}$ and the proof of the theorem is complete. ∎

3. PROOF OF THEOREM 1.1

Denote by $\overline{g} = (\overline{g}_1, \dots, \overline{g}_n)$ a smooth continuation of vector-function g into $\overline{\mathcal{D}}$ such that $\overline{g} \in C^{2,\alpha}(\overline{\mathcal{D}})$.

Let $\{\mathcal{D}_{\delta_k}\}, k = 1, 2, \dots,$ be the set of the domains such that $\mathcal{D}_{\delta_k} \subset \mathcal{D}_{\delta_{k+1}}$ and $\lim\limits_{k\to\infty} \delta_k = 0$ and let us consider the following Dirichlet problem:

$$Lu = f(x) \text{ in } \mathcal{D}_{\delta_k}, \quad u(x) = \overline{g}(x) \text{ on } \mathcal{D}_{\delta_k}. \tag{3.1}$$

Observe, that according to (1.2) system (1.1) is uniformly elliptic in $\mathcal{D}_\delta$. Since its coefficients and the right-hand side are from $C^{0,\alpha}(\overline{\mathcal{D}}_{\delta_k}) \subset C^{0,\alpha}(\overline{\mathcal{D}})$, and $\overline{g} \in C^{2,\alpha}(\overline{\mathcal{D}}_{\delta_k}) \subset C^{2,\alpha}(\overline{\mathcal{D}})$, due to lemma 2.1 there exists the unique solution $u^{(k)} = (u_1^{(k)}, \dots, u_m^{(k)}) \in C^{2,\alpha}(\overline{\mathcal{D}}_{\delta_k})$ of problem (3.1) [1], [6]. Moreover, in view of (2.1) the following estimate holds:

$$\left|u_j^{(k)}\right|_{0;\mathcal{D}_{\delta_k}} \le \max_{1\le i\le m}\{|\overline{g}_i|_{0;\mathcal{D}}, \nu^{-1}|f_i|_{0;\mathcal{D}}\}, \quad j = \overline{1,m}. \tag{3.2}$$

Let us build the set $\{u^{(k)}\}, k = 1, 2, \dots,$ whose members are the solutions of Dirichlet problem (3.1) in $\mathcal{D}_{\delta_k}, k = 1, 2, \dots,$ respectively. Now let us fix a domain $\mathcal{D}_\delta$ with arbitrary δ. Evidently, all the members of the set $\{u^{(k)}\}$ with k such that $\delta_k < \delta/2$, are defined both in $\mathcal{D}_{\delta/2}$ and in $\mathcal{D}_\delta$. Using the well known *a priori* estimates [1], [6] we obtain for these $u^{(k)}$ the estimate

$$|u_j^{(k)}|_{2,\alpha;\mathcal{D}_\delta} \le N \sum_{i=1}^{m} \Big(|\overline{g}_i|_{2,\alpha;\mathcal{D}} + |u_i^{(k)}|_{0;\mathcal{D}_{\delta/2}} + |f_i|_{0,\alpha;\mathcal{D}}\Big), \quad j = \overline{1,m}, \tag{3.3}$$

with a positive constant N independent of k. Since $|u_i^{(k)}|_{0;\mathcal{D}_{\delta/2}} \le |u_i^{(k)}|_{0;\mathcal{D}_{\delta_k}}$ for $\delta_k < \delta/2$, it follows from (3.3) due to (3.2) the compactness of the set $\{u^{(k)}\}$ in the space $C^2(\overline{\mathcal{D}}_\delta)$. Therefore, using the diagonalization method one cane choose a subset $\{u^{(k_i)}\} \subset \{u^{(k)}\}$ which strongly converges (in the sense of convergence of each component of vector-function $u^{(k_i)}$) in $C^2(\overline{\mathcal{D}}_\delta)$ to some limiting vector-function $u = (u_1, \dots, u_m)$ uniformly bounded in domain $\overline{\mathcal{D}}_\delta$ with respect to δ. Besides that, the completeness of the space $C^{2,\alpha}(\overline{\mathcal{D}}_\delta)$ implies the imbedding $u = C^{2,\alpha}(\overline{\mathcal{D}}_\delta)$, because $u^{(k_i)} \in C^{2,\alpha}(\overline{\mathcal{D}}_\delta)$ for each $k_i < \delta/2$. Hence, $u \in C_1^{2,\alpha}(\mathcal{D})$ and due to arbitrality of δ vector-function u satisfies system (1.1) everywhere in $\mathcal{D}_0$.

Observe, that the validity of (1.6) follows from uniform convergence of subset $\{u^{(k_i)}\}$ in $\overline{\mathcal{D}}_\delta$ due to condition $u^{(k_i)}\big|_S = g$ for each $i = 1, 2, \dots$

Thus, the existence of the solution of problem (1.5) – (1.7) is proved.

The uniqueness of the solution follows from lemma 2.1. Indeed, if both u^1 and u^2 are the solutions of problem (1.5) – (1.7), then $u = u^1 - u^2$ satisfies in $\mathcal{D}_0$ the equation $Lu = 0$ and boundary value condition $u\big|_S = 0$, i.e. $u \equiv 0$ in $\mathcal{D}_0$ according to theorem 2.1. Thus the proof of theorem 1.1 is complete.

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APIE IŠSIGIMSTANČIŲ VIDINIAME TAŠKE ELIPSINIŲ SISTEMŲ KLASĖS PIRMĄJĮ UŽDAVINĮ

S. Rutkauskas

Nagrinėjamas silpnai susietų antros eilėes elipsinių lygčių, išsigimstančių vidiniame srities taške, Dirichle tipo uzvdavinys. Įrodyta šio uždavinio sprendinio egzistencija ir vienatis Hiolderio vektorių-funkcijų klasėje.